Learning the Parameters of Global Constraints for Medical Scheduling

Émilie Picard-Cantin, Mathieu Bouchard, Claude-Guy Quimper, and Jason Sweeney
Émilie Picard-Cantin
**Medical Schedule**

### People oriented

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<thead>
<tr>
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### Shift / Task oriented

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Scheduling Process

Expert → Model → Solver → Schedule
Scheduling Process

- Expert
- Model
- Solver
- Schedule
Scheduling Process

Expert → Model → Solver → Schedule
Speeding up the modelling process
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• **Challenge**: Models differ from one medical team to another
Speeding up the modelling process

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• **Observation**: There are few differences between each model.
Speeding up the modelling process

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• **Opportunity**: For legal reasons, hospitals keep a history of their schedules.
Speeding up the modelling process

- **Challenge**: Models differ from one medical team to another

- **Observation**: There are few differences between each model.

- **Opportunity**: For legal reasons, hospitals keep a history of their schedules.

- **Goal**: To learn the models from historical data.
Recommander System

Past schedules
Recommander System

Recommander System  Past schedules

[Diagram showing a gear and a database]
Recommander System

Discovered Constraints  Recommander System  Past schedules
Recommander System

Discovered Constraints → Recommender System → Past schedules

← Expert → Model → Solver → Schedule
Recommander System

Discovered Constraints → Recommander System → Past schedules

Holy Grail

Expert → Model → Solver → Schedule
How to learn a constraint?

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- Do we have a limit of:
  - 2 night-shifts per week?
  - 1 night-shift every 3 days?
Which constraint was imposed?
Which constraint was imposed?

\[ x + y \leq 4 \]
Which constraint was imposed?

\[ x + y \leq 5 \]
\[ x + y \leq 4 \]
Problem Definition

- Consider a random assignment $\tilde{X}$ with $P[X_i = v] = p_v$
Problem Definition

• Consider a random assignment $\vec{X}$ with $P[X_i = v] = p_v$

• The probability of observing $\vec{X}$ is $P(\vec{X}) = \prod_{i=1}^{n} p_{X_i}$
Problem Definition

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- Consider the constraint $C([X_1, \ldots, X_n],[\alpha_1, \ldots, \alpha_m])$
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• The probability that a random assignment satisfies $C$ is

$$G_C(\tilde{\alpha}) = \sum_{\tilde{X} \mid C(\tilde{X}, \tilde{\alpha})} P(\tilde{X})$$
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• The probability that a random assignment satisfies $C$ is

  $$G_C(\tilde{\alpha}) = \sum_{\tilde{X} | C(\tilde{X}, \tilde{\alpha})} P(\tilde{X})$$

• Finding $\tilde{\alpha}$ consists in solving:

  $$\min_{\tilde{\alpha}} G_C(\tilde{\alpha})$$

  $$C(\tilde{X}, \tilde{\alpha}) \quad \forall \tilde{X} \in \text{Examples}$$
How to compute $G_C(\vec{\alpha})$?

- Enumerating and summing the probability of all solutions of a constraint is slow.

- We mainly developed two techniques to compute or bound this probability
  - Using Markov chains
  - Using dynamic programming
Markov Chains

- Some constraints can naturally be encoded with an automaton.

\[
\text{\textsc{sequence}}([X_1, \ldots, X_n], \{1\}, 0, 2, 3)
\]

1 = a night shift

at least 0

every 3 days

at most 2
Sequence Automaton

The sequence automaton is a finite state machine that processes sequences of 0s and 1s. It transitions between states based on the input symbols. The automaton starts in the start state, labeled '[]', and moves through states such as '[0]', '[0, 0]', '[0, 1]', '[1, 0]', '[1, 1]', and eventually reaches a reject state if the sequence does not match the accepted pattern.

- Start state: '[]'
- Accepting states: '0, 1'
- Rejection state: 'reject'

Transitions are as follows:
- From '[]' to '[0]' on input 0
- From '[0]' to '[0, 0]' on input 0, to '[0, 1]' on input 1
- From '[0, 0]' to '[1, 0]' on input 0, to 'reject' on input 1
- From '[0, 1]' to '[1, 1]' on input 0, to '[0, 1]' on input 1
- From '[1]' to '[1, 1]' on input 1
- From '[1, 0]' to '[1, 1]' on input 0
- From '[1, 1]' to 'reject' on input 1

The automaton accepts sequences that end in either '0, 1'.
Sequence Markov Chain
Computing $G_{\text{SEQUENCE}}(\vec{\alpha})$

- Let $M_{\vec{\alpha}}$ be the transition matrix of the Markov chain for the constraint with parameters $\vec{\alpha}$.

- One can compute the probability of reaching the reject state after reading $n$ characters by computing $M_{\vec{\alpha}}^n$.

- For every combination of $\vec{\alpha}$, compute $M_{\vec{\alpha}}^n$ and evaluate $G_C(\vec{\alpha})$.

- Keep $\vec{\alpha}$ that minimizes $G_C(\vec{\alpha})$. 
When parameters are sets

- If the parameter contains a set, there is an exponential number of combinations to explore.

\[
\text{Among}([X_1, \ldots, X_n], l, u, \bar{z})
\]
\[
\text{Sequence}([X_1, \ldots, X_n], l, u, w, \bar{z})
\]
\[
\text{SubSetFocus}([X_1, \ldots, X_n], l, m, \bar{z})
\]
Branch & Bound

\[
\min_{\vec{\alpha}} G_C(\vec{\alpha})
\]

\[C(\vec{X}, \vec{\alpha}) \quad \forall \vec{X} \in \text{Examples}\]

• We use this strategy to compute a bound on \(G_C(\vec{\alpha})\)

\[
p = \sum_v z_v \cdot p_v
\]

\[
G_C(\vec{\alpha}) \geq G_C(\text{ext}(\vec{\alpha}))
\]

\[
\geq \sum_{k=0}^{n} D[k, \text{ext}(\vec{\alpha})] \left( \min_p p^k (1 - p)^{n-k} \right)
\]
Branch & Bound

\[ \min_{\bar{\alpha}} G_C(\bar{\alpha}) \]

\[ \forall \vec{X} \in \text{Examples} \]

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\]

\[
C(\vec{X}, \vec{\alpha})
\]

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\]

Set parameters to extreme values (requires monotonicity)
Branch & Bound

\[
\min_{\bar{\alpha}} G_C(\bar{\alpha})
\]

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Count how many solutions exist with exactly k values in the set
Branch & Bound

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Count how many solutions exist with exactly k values in the set

Probability of a random assignment with k values in the set
Branch & Bound

\[
\min_{\tilde{\alpha}} G_C(\tilde{\alpha})
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Studied Constraints

SubSetFocus
Sequence
Among
GCC
AtMostNValue
AtLeastNValue
AtMostBalance
AtLeastBalance
Experiments

Table 1: Results for SUBSETFOCUS. Number of instances for which the initial constraint was ranked first, second, third or was not found.

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<th>Num. of examples</th>
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<th>Num. of instances</th>
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Conclusion

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• The system is not used!
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• The system is not used!

• It could have saved hundreds of hours in expert time.