Acquiring Maps of Interrelated Conjectures on Sharp Bounds

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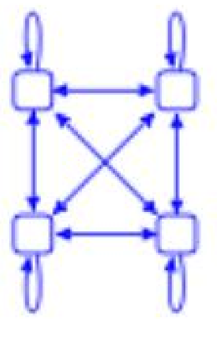
ABSTRACT

To automate the discovery of conjectures on combinatorial objects, we introduce the concept of a map of sharp bounds on characteristics of combinatorial objects, that provides a set of interrelated sharp bounds for these combinatorial objects. We then describe a Bound Seeker, a CP-based system, that gradually acquires maps of conjectures. The system was tested for searching conjectures on bounds on characteristics of digraphs: it constructs sixteen maps involving 431 conjectures on sharp lower and upper-bounds on eight digraph characteristics.

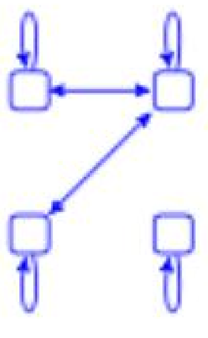
PART I: CONTEXT AND QUESTIONS

COMBINATORIAL OBJECTS AND THEIR CHARACTERISTICS

DIGRAPHS



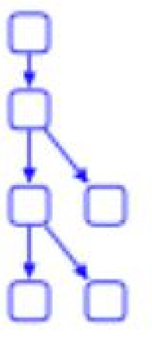
$a = 16$
$v = 4$
$c = 1$



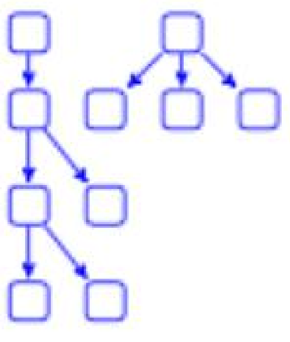
$a = 8$
$v = 4$
$c = 2$

a = number of arcs
v = number of vertices
c = number of connected components

FORESTS



$f = 3$
$t = 1$
$\overline{p} = 3$
$v = 6$



$f = 6$
$t = 2$
$\overline{p} = 3$
$v = 10$

f = number of leaves
$\overline{p}$ = maximum depth
v = number of vertices
t = number of trees

QUESTIONS

How to acquire from data representing instances of combinatorial objects:

(1) Sharp bounds of characteristics ?

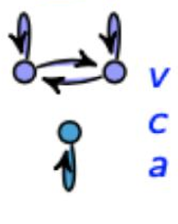
(2) Relations between sharp bounds ?

(3) And organise (1) and (2) into a map ?

Illustrating the questions wrt digraphs

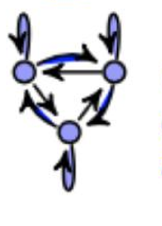
(1) Sharp bounds:

$a \leq (v-(c-1))^2 + (c-1)$



$v = 3$
$c = 2$
$a = 5$

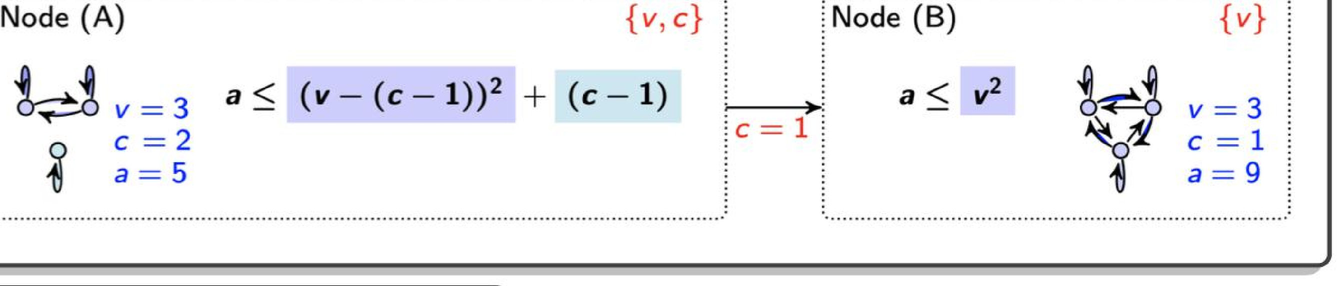
$a \leq v^2$



$v = 3$
$c = 1$
$a = 9$

(2) Relation: If $a = v^2$ then $c = 1$

(3) A map:



PART II: DEFINITION OF A MAP OF CONJECTURES

Given a finite set of input characteristics $\mathcal{P}$ and an output characteristic $o \notin \mathcal{P}$, a map of sharp upper bounds $\mathcal{M}_{\mathcal{P}}^{o\leq}$ is defined as a digraph where:

- Each node of the map is associated with a subset $P \subseteq \mathcal{P}$ of input characteristics and corresponds to a maximum conjecture of the form $o \leq f(P)$, where f is a function of P.

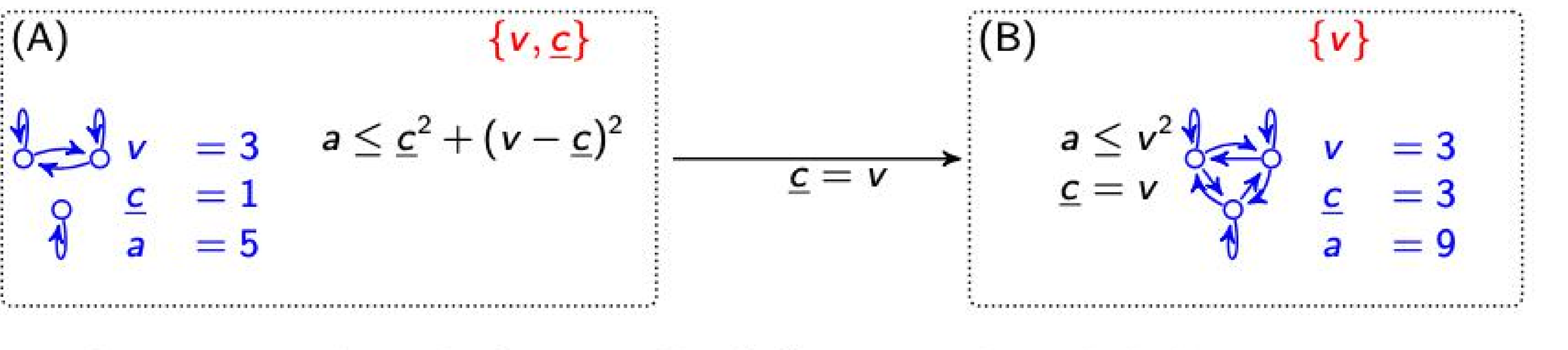
  This inequality is tight, i.e. there exist values that can be given to the parameters P in order to reach the equality.
- Each arc from conjecture $o \leq f_1(P \cup \{q\})$ to conjecture $o \leq f_2(P)$ corresponds to a projection from a subset $P \cup \{q\}$ of input characteristics to a subset P of input characteristics, by eliminating a characteristic q.

  The equality $q = g(P)$ is called a maximality conjecture.

ILLUSTRATING THE DEFINITION OF A MAP

$o \leq f_1(P \cup \{q\})$ → $q = g(P)$ → $o \leq f_2(P)$

maximum conjecture 1 — maximality conjecture — maximum conjecture 2



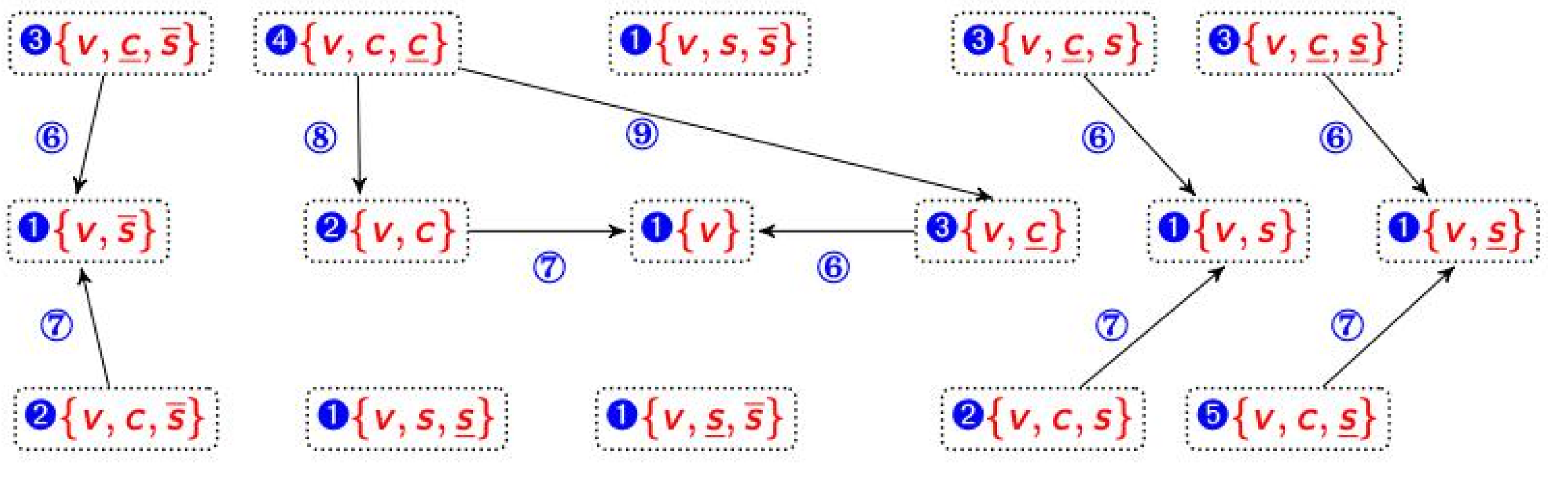
f_1 = upper bound of o wrt $P \cup \{q\}$
P = set of bounding characteristics
q = linking characteristic
o = bounded characteristic
f_2 = upper bound of o wrt P
g = a function of P

PART III: RESULTS

Comparison between the Bound Seeker (BS) and the bounds of the Global constraints catalog (GCC)

Number of bounding characteristics	1	2	3	Total	Percentage
Equivalent sharp bounds retrieved by BS	22	14	4	40	**66,66%**
Sharper bounds than the GCC found by BS	1	3	0	4	**6,66%**
Generalised sharp bounds found by BS	0	6	0	6	**10%**
Erroneous bounds found in the GCC by BS	1	1	1	3	**5%**
Bounds in the GCC not retrieved by BS	0	0	7	7	**11,66%**
Total bounds of the GCC for each column	24	24	12	60	

Map of 16 sets of bounding characteristics used to bound the characteristic $\overline{c}$



$\overline{c}$: size of the largest connected component
v : number of vertices
c : number of connected components
s : number of strongly connected components
$\underline{c}$: size of the smallest connected component
$\overline{s}$: size of the largest strongly connected component
$\underline{s}$: size of the smallest strongly connected component

❶ $\overline{c} \leq v$
❷ $\overline{c} \leq v - c + 1$
❸ $\overline{c} \leq (v = \underline{c}\ ?\ v : v - \underline{c})$
❹ $\overline{c} \leq \underline{c} - c \cdot \underline{c} + v$
❺ $\overline{c} \leq \underline{s} - c \cdot \underline{s} + v$
⑥ $\underline{c} = v$
⑦ $c = 1$
⑧ $\underline{c} = (c = 1\ ?\ v : 1)$
⑨ $c = 1 + (v \neq \underline{c})$