Dynamic Programming for the Fixed Route Hybrid Electric Aircraft Charging Problem*

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Abstract. Air mobility is rapidly moving towards the development and usage of hybrid electric aircraft in multi-flight missions. Aircraft operators must consider numerous infrastructure and operational constraints in their planning, during which predicting energy usage is critical. We introduce this problem as the *Fixed Route Hybrid Electric Aircraft Charging Problem* (FRHACP). Given a fixed route, this problem aims to decide how much to refuel/charge at each terminal as well as the energy types to use during each flight leg (hybridization). The objective is to minimize the total energy-related monetary costs while satisfying scheduling and hybridization constraints. We propose a dynamic programming algorithm to solve this problem and show that it is optimal under assumptions usually satisfied in real-life settings. We then propose a gradient descent post-treatment to relax one of these assumptions while maintaining optimality. Results on realistic instances demonstrate that the developed algorithms outperform greedy heuristics, reaching an average cost reduction of up to 19.4%.

Keywords: Energy Management · Hybrid Electric Aircraft · Air Mobility · Dynamic Programming · Optimization · FRVCP

1 Introduction

Air mobility traditionally involves aircraft powered by combustion engines using carbon-based fuels. In the past years, the interest in alternative propulsion engines significantly increased with the general aim of reducing aircraft greenhouse gas emissions. For that purpose, electricity-powered aircraft have been

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proposed, including hybrid electric aircraft which combines internal combustion engines with electrical power sources. It is envisioned that the future of air mobility will include these aircraft in a significant number of multi-flight missions, even possibly on demand, of varying length and duration [3].

Many challenges arise from the use of electricity as propulsion energy. Not only must one determine the trajectory of the vehicle, but also manage its energy consumption over the whole mission course according to aircraft, infrastructure, security, and schedule specifications. Given a flight route, this management aspect is particularly important in a planning perspective, since charging currently requires a non-negligible and non-linear amount of time [6,15]. Aircraft operators must thus decide how much to refuel and charge at each mission terminal. Furthermore, the consideration of hybrid electric aircraft requires hybridization decisions on the energy types to use (fuel and/or electricity) during each flight leg. These decisions must take into account consumption predictions from non-linear energy models [7,17], as well as mass variations and schedule requirements, to globally minimize energy-related monetary costs.

In this paper, we introduce the above-described optimization problem as the *Fixed Route Hybrid Electric Aircraft Charging Problem* (FRHACP) and propose a Dynamic Programming (DP) algorithm to solve it. Section 2 describes the FRHACP. Section 3 relates this problem to other work in the literature, notably the FRVCP for electric vehicles [7,15]. The DP algorithm is presented in Section 4, including details on the assumptions to guarantee its optimality, as well as a post-treatment to relax one of these assumptions. The algorithms are validated and compared to greedy heuristics on realistic instances in Section 5, while we conclude in Section 6.

2 The Fixed Route Hybrid Electric Aircraft Charging Problem (FRHACP)

The FRHACP considers hybrid electric aircraft in a multi-flight mission setting. A mission is defined as a fixed route $r \coloneqq (n_1, n_2, \dots, n_{|N|})$ of subsequent *nodes* $n_i \in N$. Each nodes from the route is either a *terminal* from set T or a *waypoint* from set W ($N \coloneqq T \cup W$). A terminal is typically an airport, where facilities are available to refuel and charge the aircraft. The route r starts and ends at a terminal, i.e. $n_1, n_{|N|} \in T$, while r induces a natural order $t_1, t_2, \dots, t_{|T|}$ on the terminals in T. Between consecutive terminals, the route is defined by waypoints, typically reference points in the air that must be part of the aircraft trajectory. We define *legs* as route segments connecting two consecutive nodes such as $L \coloneqq \{(n_i, n_{i+1}) : i = 1, \dots, |N|-1\}$.

The FRHACP asks to decide how much to refuel and charge the aircraft at each terminal. Fuel quantity in the aircraft is limited by a minimal security margin f^{min} and the tank capacity f^{max}. Similarly, the aircraft battery State of Charge (SoC) is limited by minimal and maximal security margins, s^{min} and s^{max}. Each terminal $t \in T$ is also associated with a scheduled departure time d_t^{time} to respect as a hard constraint. The time needed to charge the battery from SoC s_1 to s_2 at terminal $t \in T$ is predicted with $\alpha_t^s(s_1, s_2)$, usually non-linear [6,15]. Refueling duration is given by a constant rate α^f depending on quantity.

Hybridization decisions on the energy types to use (fuel and/or electricity) during each leg are also part of the FRHACP. On that matter, it is known in the literature that the optimal energy management strategy on a leg is to use the fuel first, then the electricity [16]. Furthermore, fuel has a non-negligible mass, here encoded as a constant ratio of m_f depending on volume. This is known to be an important non-linear factor impacting the fuel and electricity consumption [17]. Thus, this problem encodes hybridization on each leg as a percentage of its distance using fuel, while the remaining distance is done using electricity, with fuel used first. Aircraft mass, m_a, and payload mass at terminal t, m_t^p, are also considered. Fuel and electricity consumption prediction models are encoded as functions dependant on the travel distance d and the total mass m, denoted respectively by $\delta^f(d, m)$ and $\delta^s(d, m)$. These functions are usually based on non-linear energy models including numerous other physical parameters [7,17] that are assumed constant on a given leg, but allowed to vary between legs (e.g., speed, altitude, and trajectory angle).

We resume the decisions variables of this problem as follows. For each terminal $t \in T$, $F_t^D \in [f^{min}, f^{max}]$ and $S_t^D \in [s^{min}, s^{max}]$ are respectively the fuel quantity and SoC of the aircraft when departing from terminal t. Then, for each leg $l \in L$, $H_l \in [0, 1]$ is the hybridization on leg l as its percentage traveled using fuel. Intermediate variables F_t^A and S_t^A describe the deduced fuel quantity and SoC upon arrival at terminal $t \in T$.

Finally, each terminal $t \in T$ has a fuel (resp. electricity) cost c_t^f (c_t^s) per refueled (charged) quantity. The FRHACP objective is thus to minimize the mission total cost according to energy decisions, i.e.

$$\min \sum_{t \in T} \left(c_t^f \left(F_t^D - F_t^A \right) + c_t^s \left(S_t^D - S_t^A \right) \right). \tag{1}$$

3 Related Work

The FRHACP is highly related to the *Fixed Route Electric Vehicle Charging Problem* (FRVCP) introduced by Montoya *et al.* [15], which has recently been extended with non-linear energy management in the context of an electric vehicle route planning [7]. In this problem, the objective is to minimize the total route duration including its charging time by considering variable vehicle speed and charging detours, while handling the non-linearity of electricity. The FRVCP has been solved using dynamic programming [5], Mixed Integer Programming [7] and labeling algorithms [10]. The FRHACP can naturally be seen as a variant of the FRVCP adapted to the context of hybrid electric aircraft. The main differences are the hybridization decisions and the objective function.

It is also well known in the literature that the non-linearity of energy models, depending among others on vehicle specifications, speed, mass, and temperature, are essential for energy-related predictions and planning [2,4,6,17]. Prior work in the hybrid electric aircraft domain mainly relates to optimal hybrid management [16], energy architecture [14,18] and fuel/electricity consumption models for different aircraft configurations [13,19]. Notably, *OpenAP* provides open-source aircraft performance and emission models based on open data and accessible for air transport research [17].

4 Dynamic Programming Algorithm

The FRHACP defined in Section 2 is more complex than the FRVCP and its variants [5,7] since two energy sources must be simultaneously considered (fuel and electricity). It is thus harder to design a dynamic programming algorithm following the approach of Deschênes *et al.* [5]. At least one state space must be added for the fuel. This would increase the computation time based on the number of sampled fuel quantities, say $\tilde{f}$. When refueling and charging, combinations of fuel and SoC will need to be considered, say $\tilde{f} \cdot \tilde{s}$. Thus we can estimate that the algorithm would be at least $\tilde{f}^2 \cdot \tilde{s}$ times slower, without even considering the additional computation time of 2-dimensional interpolation. With $\tilde{f} = 50$ and $\tilde{s} = 10$, it would be at least 25 000 times slower. Thus, the dynamic programming curse of dimensionality quickly arises.

Nevertheless, under some assumptions, it is possible to design a dynamic programming algorithm that optimally solves the problem. This algorithm is presented in Section 4.1. In Section 4.2, we develop a gradient descent post-treatment that allows to relax one of these assumptions while maintaining optimality.

4.1 Minimizing Total Cost

The proposed approach looks at the total cost minimization problem from the perspective of minimizing the fuel quantity in a number of subproblems. Each flight between consecutive terminals t_i and t_{i+1} defines a different subproblem, leading to the following question for all nodes n_k between t_i and t_{i+1} inclusively: *Given a current SoC s, what is the minimal fuel quantity $F^*_{n_k}(s)$ needed to reach terminal t_{i+1} from node n_k while satisfying all constraints?* Equation (2) presents the recurrence used to answer this question.

$$F^*_{n_k}(s) = \begin{cases} f^{min} & \text{if } n_k = t_{i+1} \\ \min\limits_{h \in [0,1]} \left[F^*_{n_{k+1}}(s - \Delta^s_{l_k}(h)) + \Delta^f_{l_k}(h) \right] & \text{otherwise} \end{cases} \quad (2)$$

If node n_k is terminal t_{i+1}, the minimal quantity to reach itself is trivially the margin f^{min}. Otherwise, the minimal quantity from n_k depends on the hybridization decision $h \in [0,1]$ on leg $l_k := (n_k, n_{k+1})$. Here, we respectively denote $\Delta^f_{l_k}(h)$ and $\Delta^s_{l_k}(h)$ the fuel and electricity consumption on leg l_k given h. The SoC at node n_{k+1} is thus given by $s - \Delta^s_{l_k}(h)$, while the minimal fuel quantity needed at n_{k+1} is $F^*_{n_{k+1}}(s - \Delta^s_{l_k}(h))$. We must then add the amount of fuel needed on leg l_k as given by $\Delta^f_{l_k}(h)$. Taking the minimal value over all h, $F^*_{n_k}(s)$ returns the minimal fuel quantity from n_k. Proposition 1 directly follows from this inductive reasoning.

Proposition 1. *The problem of minimizing the fuel quantity between consecutive terminals t_i and t_{i+1} admits an optimal substructure. In other words, given a current SoC s, $F^*_{n_k}(s)$ is the minimal fuel quantity to reach t_{i+1} from node n_k for all $n_k \in N$ between t_i and t_{i+1}.*

To solve the recurrence, we compute $\tilde{h}$ values of h and take the minimal computed quantity. $\Delta^f_{l_k}(h)$ is obtained with $\delta^f(h \cdot d_k, m(h))$ where d_k is the total distance of leg l_k and $m(h)$ is the mass, considering m_a, $m^p_{t_i}$, and the fuel mass in the tank depending on h. Similarly, $\Delta^s_{l_k}(h)$ is obtained with $\delta^s((1-h) \cdot d_k, m(h))$. As in Deschênes *et al.* [5], the state space of s is continuous, thus we use the same techniques to solve the problem. We sample $F^*_{n_k}(s)$ for $\tilde{s}$ different SoC s for each node n_k. Finally, we use Akima interpolation [1] to approximate the overall function $F^*_{n_k}(s)$ for each node n_k.

Constructing the route solution. Our proposed DP algorithm solves the problem by constructing decisions for the complete route. It first starts at terminal t_1 with the initial fuel and SoC of the aircraft. Then, it finds $S^D_{t_1}$ satisfying the schedule $d^{time}_{t_1}$ and margin s^{max} using a binary search. From $S^D_{t_1}$, it uses recurrence (2) to compute $F^D_{t_1}$ and follows it until reaching terminal t_2. Note that it always makes sure f^{min}, f^{max}, and s^{min} are satisfied. The hybridization decisions H_{l_k} on legs l_k between t_1 and t_2 are simultaneously computed by the recurrence (as the arg min). At each following terminal $t_i \in T$, we determine $S^D_{t_i}$ and $F^D_{t_i}$ in the same way. This gives us our final solution. In order to prove this solution is optimal when minimizing the total cost under some assumptions, the following definition is needed.

Definition 1 (Independence of subproblems). *All subproblems are independent if $F^A_t = f^{min}$ and $S^A_t = s^{min}$ $\forall t \in T$ in the optimal solution.*

The independence of subproblems is known to imply *at least* these necessary conditions: (1) Fuel cost c^f_t is the same at each terminal $t \in T$; (2) Between each consecutive terminal, the optimal solution consumes all fuel and electricity. Note that other conditions might be needed to fully ensure independence of subproblems on some instances. For example, it is possible to construct an instance where d^{time}_t constrains the charging time in a way that electricity must be stored from a previous terminal, violating the independence while satisfying the above-mentioned conditions.

Proposition 2. *Suppose that consumption functions $\delta^f(d,m)$ and $\delta^s(d,m)$ are monotonically increasing with respect to d and m, that we have independence of subproblems, and that electricity costs c^s_t are significantly lower than fuel costs c^f_t. Then the DP constructed solution optimally minimizes total cost.*

Proof. With independence of subproblems, the constructed solution is such that all fuel and electricity is consumed between all consecutive terminals. According to Equation (1) and the assumption about electricity costs, the only way to reduce the total cost would be to reduce the fuel consumption in each subproblem. Since $\delta^f(d,m)$ and $\delta^s(d,m)$ are monotonically increasing (more fuel leads to more mass, increasing the overall consumption), reducing the consumption is only possible by reducing the fuel quantity. However, by Proposition 1, this quantity is already minimal. □

Complexity analysis. Suppose the calls to $\delta^f(d,m)$ and $\delta^s(d,m)$ consumption functions are executed in constant time. To solve the problem, we compute the recurrence for $\tilde{s}$ values of SoC for each node $n_k \in N$. Computing $F^*_{n_k}(s)$ has a time complexity of $\Theta(\tilde{h})$ testing $\tilde{h}$ hybridization decisions. Since computing the Akima interpolation is done in a linear time, the overall time complexity is $\Theta(\tilde{s} \cdot \tilde{h} \cdot |N|)$. Thus, the algorithm running time increases *pseudo-linearly* with the number of nodes in the route.

4.2 Gradient Descent Post-Treatment

Most of the assumptions of Proposition 2 are usually satisfied in real-life settings, except for the implied condition that fuel cost is the same at each terminal. Algorithm 1 relaxes independence of subproblems by allowing $F^A_t > f^{min}$ at all terminals $t \in T$.

The algorithm starts by computing the DP solution before improving it further with a gradient descent. The problem is encoded as a directed graph $G = (T, E)$, where $E \coloneqq \{(t_i, t_j) : t_i, t_j \in T, i < j, c^f_{t_i} < c^f_{t_j}\}$. It defines the possibilities of *transferring* fuel between terminals t_i and t_j to save on fuel costs. The action of transferring x quantity of fuel from terminal t_j to t_i, denoted Transfer(x, t_i, t_j), ensures that $F^A_{t_j}$ is increased by x. It is achieved by increasing $F^D_{t_i}$ by *at least* x. The action takes into consideration the non-linearity of $\delta^f(d,m)$ and $\delta^s(d,m)$, i.e. that taking more fuel at terminal t_i will increase the mass and thus the energy consumption until we reach terminal t_j. It takes into account the fact that we may need to add more fuel to compensate for the mass increase or the hybridization correction on the legs to ensure electricity margins. Indeed,

Algorithm 1 Gradient Descent Post-Treatment (DP+GD)

1: Compute a solution using the DP algorithm
2: Construct the directed graph $G = (T, E)$, $E \coloneqq \{(t_i, t_j) : t_i, t_j \in T, i < j, c^f_{t_i} < c^f_{t_j}\}$
3: Initialize gradients $g_e \leftarrow 1, \forall\, e \in E$
4: **while** $\exists e \in E$ such that $g_e > 0$ **do**
5: Compute g_e for each edge $e \in E$
6: Find $(t_i, t_j) \in E$ such that $g_{(t_i,t_j)}$ is maximal
7: **if** $g_{(t_i,t_j)} > 0$ **then** Transfer$(\alpha \cdot g_{(t_i,t_j)}, t_i, t_j)$
8: **return** The updated solution

the latter is due to the fact that, because of the schedule, we cannot take more electricity to compensate for the increase in electricity consumption, thus we instead increase the fuel consumption by modifying the hybridization decisions. Finally, the action is only possible if the transfer allows to satisfy f^{max} and $d_{t_i}^{time}$.

The algorithm computes the *gradient* $g_{(t_i,t_j)}$ of each edge $(t_i, t_j) \in E$, i.e. how much a small transfer of fuel from terminal t_j to t_i changes the overall cost of the solution. We then do a gradient descent to transfer fuel on the maximum gradient edge. These transfers are repeated until we reach convergence, i.e. when all gradients are non-positive. If the maximal gradient is $g_{(t_i,t_j)} > 0$, we do $\textsc{Transfer}(\alpha \cdot g_{(t_i,t_j)}, t_i, t_j)$ to transfer the fuel, where α is a strictly positive learning rate. Thus, at each iteration, the solution changes. Since by definition the graph is acyclic and we can only transfer in the direction of the edge (i.e. not backwards), the algorithm terminates in a finite number of steps.

Proposition 3. *Suppose the assumptions of Proposition 2 where we relax the independence of subproblems by allowing $F_t^A > f^{min}$ at all terminals $t \in T$. Then Algorithm 1 converges to the global optimum.*

Proof. Let c_a^* be the cost of solution a returned by Algorithm 1. Suppose the contrary, i.e. that there exists a solution π following the assumptions with total cost $c_\pi^* < c_a^*$. By Proposition 2, we know that the decrease in cost cannot be from using less fuel or using the electricity. Thus, the only option would be by exploiting the relaxed assumption. This implies that solution π takes more fuel at least at one terminal to reduce the total cost, thus that there exist terminals t_i and t_j with $c_{t_i}^f < c_{t_j}^f$ not exploited by solution a. By construction of Algorithm 1, this leads to an edge (t_i, t_j) with gradient $g_{(t_i,t_j)} > 0$. However, this is impossible since the algorithm terminates with all gradients non-positive. □

5 Experiments

The main goal of the experiments is to compare the proposed DP algorithms with heuristics on real-life inspired instances. It aims at showing the benefits of using the electric engine, while doing optimized refueling, charging, and hybridization decisions.

Fuel First Heuristic (FF-H). This heuristic aims at globally maximizing the fuel usage during the flight route. It imposes a hybridization decision of 100% fuel ($H_l \coloneqq 1$) on each leg. Then, the departure fuel F_t^D is adjusted to minimize the consumption while satisfying the minimal margin f^{min}. Summarized steps of this heuristic are presented in Algorithm 2.

Algorithm 2 Fuel First Heuristic (FF-H)

1: Initialize $F_t^D \leftarrow f^{max}$ and $S_t^D \leftarrow s^{min}$ for $t \in T$; $H_l \leftarrow 1$ for $l \in L$
2: Compute F_t^A for $t \in T$
3: For $t_i \in T, i = 1, \ldots, |T|-1$, correct $F_{t_i}^D$ so that $F_{t_{i+1}}^A = f^{min}$

Maximize Battery Usage Heuristic (MB-H). This heuristic aims at globally maximizing the electricity usage during the flight route. It tries to impose a hybridization decision of 100% electricity ($H_l \coloneqq 0$) on each leg. This is often impossible, thus it handles these cases based on a greedy hypothesis of using the fuel first. The minimal quantity of fuel is computed so that the arrival SoC S_t^A reaches the margin s^{min} for $t \in T$. Summarized steps of this heuristic are presented in Algorithm 3.

Algorithm 3 Maximize Battery Usage Heuristic (MB-H)

1: Initialize $F_t^D \leftarrow f^{min}$ for $t \in T$
2: Set S_t^D to its maximal value given d_t^{time} for $t \in T$; $H_l \leftarrow 0$ for $l \in L$
3: Compute S_t^A for $t \in T$
4: **for all** $t \in T$ **where** $S_t^A < s^{min}$ **do**
5: Set F_t^D to the minimal fuel quantity satisfying s^{min}, with H_l using fuel first
6: For $t_i \in T, i = 1, \ldots, |T|-1$, correct $S_{t_i}^D$ so that $S_{t_{i+1}}^A = s^{min}$

5.1 Experimental Setup

We implemented the algorithms described in Section 4 and the heuristics in Python. The experiments were performed on an Intel Core i7-8750H CPU @ 2.20 GHz, 6 cores and 8 GB of RAM. The DP algorithm has two different hyperparameters affecting the quality of the solution and its computation time: $\tilde{h}$, the number of hybridization values tested on each leg, and $\tilde{s}$, the number of SoC values sampled to determine the minimal fuel. For our experiments, we used $\tilde{h} = 40$ and $\tilde{s} = 10$. These values were empirically determined to yield good results in a decent computation time. For the DP+GD algorithm, the gradient is approximated by forward difference. We also used a constant learning rate $\alpha = 500$ empirically determined for converging quickly.

5.2 Instances

Our dataset consists of four real-life inspired instances, created from day-long sequences of commercial flights with the same aircraft in Canada and France. The routes and their schedule are generated using available data in FlightRadar24 [9], while the fuel and electricity costs are directly taken from various credible sources [8,11,12]. For the purpose of comparison, we convert EUR (€) costs in CAD ($) using 1.49 as exchange rate. Table 1 presents the particularities of each instance. In France, $\texttt{PN}_{T4W59}$ describes two round trips between Paris and Nice, while $\texttt{TB}_{T5W42}$ includes a round trip from Toulouse to Lille followed by a round trip from Bordeaux to Marseille. In Canada, $\texttt{MS}_{T6W30}$ includes a flight from Montreal to Quebec City, followed by a round trip to the Magdalen Islands, then a flight to Sept-Îles. $\texttt{OT}_{T7W43}$ describes a flight from Ottawa to Toronto, followed by a round trip to St. John's, Newfoundland.

Table 1. Description of the four real-life inspired instances forming the dataset.

Instance	$\|T\|$	$\|W\|$	Duration	Distance (km)	c^s ($/kWh)	c^f ($/L)
$\texttt{PN}_{T4W59}$	4	59	4h30	2740	0.1397	1.46
$\texttt{TB}_{T5W42}$	5	42	6h42	2812	0.1397	1.46
$\texttt{MS}_{T6W30}$	6	30	7h22	2294	0.0533	[1.16, 1.25]
$\texttt{OT}_{T7W43}$	7	43	9h34	4709	[0.0533, 0.1140]	[1.03, 1.28]

All instances suppose a *Cessna S550 Citation II* as the aircraft. Following the approach of Wang *et al.* [18], we suppose a battery of 216 kWh with a mass of 600 kg, giving a total mass m_a of 4256 kg. We also suppose a payload m_t^p varying at terminals $t \in T$ between 400 kg and 800 kg. The Cessna uses Jet-A1 fuel with ratio m_f 0.819 kg/L, a fuel capacity f^{max} of 3260 L, and a refueling rate α^f of 1086 L per minute. In addition, the following security margins are considered: $f^{min} = 163$ L (5%), $s^{min} = 10\%$, and $s^{max} = 95\%$.

We use OpenAP aircraft performance model [17] to predict the fuel consumption as function $\delta^f(d, m)$. To do so, we deduce altitude, distance, speed, and trajectory angle from the instance generated route. We also suppose a cruise phase at an altitude of 10.7 km with a speed of 777 km/h. All other parameters are implicitly encoded in the OpenAP model. For predicting the electricity consumption $\delta^s(d, m)$, we use OpenAP predicted net thrust and convert it to kWh.

For the charging time prediction $\alpha_t^s(s_1, s_2)$, we use for all terminals $t \in T$ the non-linear charging function from Deschênes *et al.* [7]. Although this function is unrealistic given that it has been designed for a 40 kWh battery of an electric vehicle, we envision that charging technology in a near future may allow similar durations.

5.3 Results

Table 2 presents the results of our experiments. For each algorithm (DP, DP+GD, FF-H, MB-H) and each instance, we report the solving time, as well as costs and consumed quantities related to each energy type. We also distinguish the solving time of the algorithms (*internal*) from the calls to OpenAP performance model

Table 2. Solving time of each instance in seconds — distinguished between internal (algorithms) and external (OpenAP) time — as well as costs and consumed quantities for fuel and electricity. Results reported for the Dynamic Programming algorithm (DP), DP with the Gradient Descent post-treatment (DP+GD), the Fuel First heuristic (FF-H) and the Maximum Battery heuristic (MB-H).

Instance	Algorithm	Solving time (s)			Costs ($)			Consumption	
		Internal	External	Total	Fuel	Elec.	Total	Fuel (L)	Elec. (kWh)
$\texttt{PN}_{T4W59}$	DP	0.43	4.65	5.08	4284	148	**4433**	2930	712
	DP+GD	-	-	-	-	-	-	-	-
	FF-H	0.06	0.06	0.12	5316	0	5316	3634	0
	MB-H	0.40	0.55	0.95	4579	128	4707	3131	614
$\texttt{TB}_{T5W42}$	DP	0.32	3.78	4.10	4335	142	**4477**	2991	805
	DP+GD	-	-	-	-	-	-	-	-
	FF-H	0.05	0.05	0.10	5421	0	5421	3740	0
	MB-H	0.26	0.40	0.66	4630	126	4756	3195	691
$\texttt{MS}_{T6W30}$	DP	0.23	2.40	2.63	3148	28	3176	2650	899
	DP+GD	0.40	2.56	2.96	3084	28	**3112**	2659	899
	FF-H	0.02	0.02	0.04	4081	0	4081	3429	0
	MB-H	0.14	0.14	0.28	3315	24	3338	2789	798
$\texttt{OT}_{T7W43}$	DP	0.34	4.32	4.66	6449	95	6543	5408	1210
	DP+GD	1.79	5.94	7.73	6173	95	**6268**	5523	1210
	FF-H	0.05	0.05	0.10	7822	0	7822	6540	0
	MB-H	0.37	0.50	0.87	6836	83	6919	5730	1038

(*external*). The smallest total costs are in bold. Note that it is possible to check that all instances follow the assumptions discussed in Section 4. Since $\texttt{PN}_{T4W59}$ and $\texttt{TB}_{T5W42}$ have no fuel cost variation, DP is optimal for these instances by Proposition 2 and the gradient descent post-treatment is not required. On the other hand, DP+GD is optimal for $\texttt{MS}_{T6W30}$ and $\texttt{OT}_{T7W43}$ by Proposition 3.

As expected, heuristics have the smallest solving times, while all algorithms terminate within 8 seconds. We remark that the gradient descent can increase the computation time of up to 40.0% ($\texttt{OT}_{T7W43}$). On average, 92% of DP computation time comes from external calls, i.e. OpenAP. This is reduced to 80 % with DP+GD.

About costs, DP and DP+GD (when applicable) obtain the smallest total cost on all instances. The reduction mainly comes from lower fuel consumption. On instances where the fuel costs vary ($\texttt{MS}_{T6W30}$, $\texttt{OT}_{T7W43}$), the gradient descent post-treatment of DP+GD allows an average reduction of 3.1% compared to DP. As expected, the electricity costs remain constant, since the post-treatment does not affect charging decisions. FF-H obtains the highest costs on all instances, with the DP algorithms leading to a reduction of up to 23.7% (average 19.4%). This clearly shows the benefits of using the electric engine. MB-H has smaller costs compared to FF-H, but the DP algorithms can reduce them of up to 9.4% (average 7.0%). This shows that smarter hybridization and refueling decisions can lead to better solutions.

6 Conclusion

In this paper, we introduced the FRHACP, a variant of the FRVCP adapted to the context of hybrid electric aircraft. The problem aims to handle refueling/charging and hybridization decisions given a fixed route, while minimizing energy costs and satisfying various requirements, such as mass and schedule. To solve the problem, we proposed a dynamic programming algorithm that has been shown to be optimal under some assumptions. In order to fit for more real-life settings, we relaxed one of these assumptions and allowed fuel costs to vary between terminals by proposing a gradient descent post-treatment while maintaining optimality. The algorithms were compared to two greedy heuristics on four real-life inspired instances that showed the benefits of considering electric engines and doing smart hybridization decisions. Results demonstrated an average cost reduction of up to 19.4%. The proposed algorithms found the optimal solution within 8 seconds on all instances.

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