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Optimal Single- and Multiple-Tree Almost Instantaneous Variable-to-Fixed Codes

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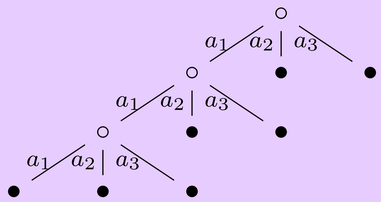
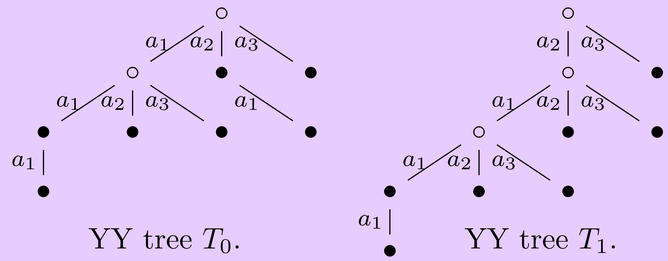
Tunstall Trees (PFVF)



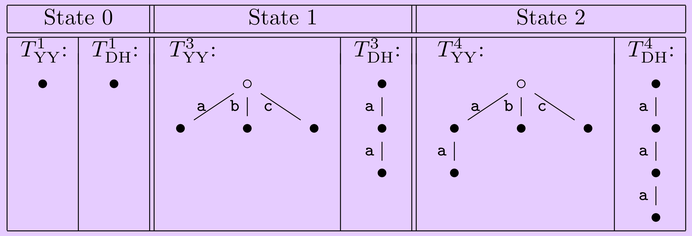
Tunstall tree T_{Tun}

Yamamoto-Yokoo Trees (AIVF)

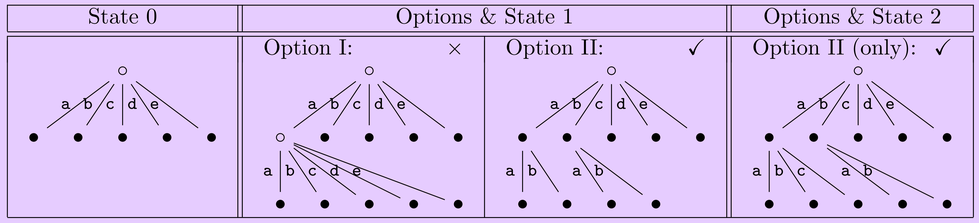


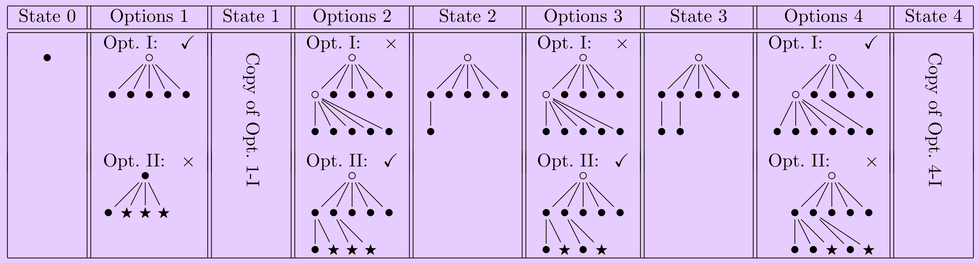
YY tree T_0. YY tree T_1.

Defect in YY Due to the Complete-Root Constraint

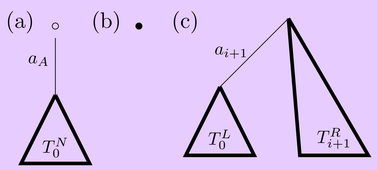


Defect in YY Due to the Full Execution of Option II





Basic Shapes of Trees Built using DP



(a) $T^N_{A-1} = \textsc{Default}(T^N_0)$
(b) $T^1_i = \textsc{Root}$, where: $i \leq A-2$
(c) $T^N_i = T^L_0 \oplus T^R_{i+1}$, where: $i \leq A-2$, $2 \leq N = L + R$

(**Figure 1.**)

Future Work

- We should verify whether applying our correctives on YY would make it optimal.
- The AI property remains a constraint on the considered VF codes, even if it is looser than the PF property, and we should investigate on the opportunities offered by the removal or relaxation of this constraint; e.g., codes with a longer delay [3].
- Related work: Almost instantaneous fixed-to-variable (AIFV) codes by Yamamoto et al [2, 3].

References

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Algorithms

Algorithm COMPLETE(t)**: add a node's missing children**

$\mathcal{V} \leftarrow \{\pi(n) \mid n$ is an incomplete nodes in $t\}$
$v_{\max} \leftarrow \arg\max_{v \in \mathcal{V}} p(v)$
$\mathcal{W} \leftarrow \{v_{\max}\} \cdot \mathcal{A}$ /* Paths to children of $v_{\max}$ */
return $t + \mathcal{W}$

Algorithm TUNSTALL(M)**: build an up-to-M-codeword tree**

Require: $M \geq A$
$t_{\text{new}} \leftarrow \textsc{Root} + \{a_1, \ldots, a_A\}$
repeat
  $t_{\text{old}} \leftarrow t_{\text{new}}$
  $t_{\text{new}} \leftarrow$ COMPLETE(t_{old})
until $\#t_{\text{new}} > M$ /* Ultimate t_{new} gets discarded */
return t_{old} /* $M - (A-1) < \#t_{\text{old}} < M$ */

Algorithm EXTEND(t)**: add the best child**

$\mathcal{U} \leftarrow \{\pi(n) \mid$ node n in $t\}$ /* Paths to all nodes */
$\mathcal{V} \leftarrow \mathcal{U} \cdot \mathcal{A}$ /* Path extensions */
$\mathcal{W} \leftarrow \mathcal{V} - \mathcal{U}$ /* Paths to potential children */
$w_{\max} \leftarrow \arg\max_{w \in \mathcal{W}} p(w)$
return $t + \{w_{\max}\}$

Algorithm YY(i, M)**: build an M-codeword T_i**

Require: $0 \leq i \leq A-2$ **and** $M \geq A - i$
$t_{\text{new}} \leftarrow \textsc{Root} + \{a_{i+1}, \ldots, a_A\}$
repeat
  $t_{\text{old}} \leftarrow t_{\text{new}}$
  $t_{\text{I}} \leftarrow$ COMPLETE(t_{old}) /* Option I */
  $t_{\text{II}} \leftarrow \text{EXTEND}^{\#t_{\text{I}} - \#t_{\text{old}}}(t_{\text{old}})$ /* Option II */
  $t_{\text{new}} \leftarrow$ the best of t_{I} and t_{II}
until $\#t_{\text{new}} > M$ /* Ultimate t_{new} gets discarded */
return $\text{EXTEND}^{M - \#t_{\text{old}}}(t_{\text{old}})$ /* Option II */

Algorithm BUILD(i, N)**: build an N-codeword T_i in a DP way**

Require: $0 \leq i \leq A-1$ **and** $N \geq 1$
if $i = A - 1$ **then**
  return $\textsc{Default}(T_0^N)$ /* Fig. 1(a) */
else if $N = 1$ **then**
  return $\textsc{Root}$ /* Fig. 1(b) */
else
  $\mathcal{T} \leftarrow \{T_0^L \oplus T_{i+1}^R \mid L + R = N,\ L \geq 1,\ R \geq 1\}$ /* Fig. 1(c) */
  return $\arg\max_{t \in \mathcal{T}}$ AvgParsewordLen(t) /* $O(N)$-time case */
end if

Algorithm FILL(M)**: build all up-to-M-codeword trees in a DP way**

Require: $M \geq 1$ /* $O(A \cdot M^2)$-time algorithm */
for $N = 1$ to M **do**
  for $i = 0$ to $A - 1$ **do**
    $T_i^N \leftarrow$ BUILD(i, N)
  end for
end for

Algorithm $\text{BUILD}^{\text{single}}$($i$, N)**: build a complete-root N-codeword T_i**

Require: $0 \leq i \leq A-1$ **and** $N \geq A - i$
if $i = A - 1$ **then**
  return $\textsc{Default}(T_0^N)$
else
  $\mathcal{S} \leftarrow \{T_0^L \oplus S_{i+1}^R \mid L + R = N,\ L \geq 1,\ R \geq 1,\ R \geq A - (i+1)\}$
  return $\arg\max_{t \in \mathcal{S}}$ AvgParsewordLen(t)
end if

Algorithm $\text{FILL}^{\text{single}}$($M$)**: build all complete-root up-to-M-codeword trees**

Require: $M \geq 1$
for $N = 1$ to M **do**
  for $i = \max(A - N, 0)$ to $A - 1$ **do**
    $S_i^N \leftarrow \text{BUILD}^{\text{single}}(i, N)$ /* S_i^N undefined for $N < A - i$ */
  end for
end for