Série 9

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Problème 1

$SNR_{lin} = 24 \rightsquigarrow SNR_{dB} = 10 \log_{10}(SNR_{lin})$

$SNR_{dB} = 10 \log_{10} 24 = 13,802$

d'un autre coté : $SNR_{dB} = 6.02\,n$ dB

$\Rightarrow 6.02\,n = 13,802 \Rightarrow \boxed{n = 3}$

Il faut concevoir un quantificateur à 3 bits.

Problème 2

a) Quatification uniforme : à 2 bits

7 4 0 6 7 6 1 0 3 2 6 4 1 0 2 5

111 010 000 000 111 110 001 000 011 010 110 100, 001, 000, 010 101

Code : 11 01 00 00 11 11 00 00 01 01 11 10 00 00 01 10

Reconstruction 110 010 000 000 110 110 000 000 010 010 110 100 000 000 010 100

6 4 0 0 6 6 0 0 2 2 6 4 0 0 2 4

$$MSE = \left(1^2 + 0^2 + 0^2 + 0^2 + 1^2 + 0^2 + 1^2 + 0^2 + 1^2 + 0^2 + 0^2 + 0^2 + 1^2 + 0^2 + 0^2 + 1^2 \right) / 16$$

$$= 6/16 = 0{,}375$$

ⓑ Quantification adaptative :

bloc 1

7	4
7	6

$V_{min} = 4$

$V_{max} = 7 \rightarrow 8 - V_{max} = 1$

$V_{min} \geq 8 - V_{max}$

envoi 0

envoi $V_{min} = 4$

I

Plage ~~Delta~~ $= 4 \rightarrow 7 \Rightarrow$ ~~[illegible]~~

Quantification 1 bit

		Reconstruction
4	→ 0	4
5	→ 0	4
6	→ 1	6
7	→ 1	6

MSE 1 ~~distorsion~~ : $(0^2 + 1^2 + 0^2 + 1^2)/16 = 2/16$

bloc 2 : MSE 2 = 0

bloc 3 : MSE 3 = 1/16

bloc 4 : MSE = 1/16.

MSE = 4/16

et R = 2 bits / valeur

Série 10

1) $$C_{XX} = \begin{bmatrix} E(X_1^2) - E(X_1)^2 & E(X_1X_2) - E(X_1)E(X_2) \\ E(X_1X_2) - E(X_1)E(X_2) & E(X_2^2) - E(X_2)^2 \end{bmatrix}$$

X : $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ et $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

(1) $$E(X_1^2) - E(X_1)^2 = \frac{2^2 + 1^2 + 3^2}{3} - \left(\frac{2+1+3}{3}\right)^2$$

$$= \frac{14}{3} - 4 = 0{,}666$$

(2) $E(X_1X_2) - E(X_1)E(X_2) =$

$$\frac{2\times 3 + 1\times 2 + 3\times 1}{3} - 2\times 2 = -0{,}333$$

(3) $E(X_2^2) - E(X_2)^2 = 0{,}666$

$$C_{XX} = \begin{pmatrix} 0{,}666 & -0{,}333 \\ -0{,}333 & 0{,}666 \end{pmatrix}$$

Normalisation $C_{XX} = \begin{pmatrix} 1 & -0{,}5 \\ -0{,}5 & 1 \end{pmatrix}$

2) Valeurs propres :

$$|\lambda I - C_{xx}| = 0 \Rightarrow \left| \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} - \begin{pmatrix} 1 & -0{,}5 \\ -0{,}5 & 1 \end{pmatrix} \right| = 0$$

$$\begin{vmatrix} \lambda - 1 & 0{,}5 \\ 0{,}5 & \lambda - 1 \end{vmatrix} = (\lambda-1)^2 - 0{,}5^2 = 0$$

$$= \lambda^2 - 2\lambda + 1 - 0{,}25 = 0$$

$$= \lambda^2 - 2\lambda + 0{,}75 = 0$$

$$\lambda_1 = 0{,}5 \quad \text{et} \quad \lambda_2 = 1{,}5$$

3) vecteurs propre. $U = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ et $V = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

$$C_{xx} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \lambda_1 \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -0{,}5 \\ -0{,}5 & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = 0{,}5 \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$u_1 - 0{,}5\, u_2 = 0{,}5\, u_1 \Rightarrow u_1 = u_2$

$-0{,}5\, u_1 + u_2 = 0{,}5\, u_2 \Rightarrow u_1 = u_2$

$\Rightarrow U = \begin{pmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{pmatrix}$

De même on trouve $V = \begin{pmatrix} \sqrt{2}/2 \\ -\sqrt{2}/2 \end{pmatrix}$

$$\Rightarrow K = 1/\sqrt{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

4) Transformation.

$$\theta_1 = K \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 2+3 \\ 2-3 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 5 \\ -1 \end{pmatrix}$$

$$\theta_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

$$\theta_3 = \frac{1}{\sqrt{2}} \begin{pmatrix} 4 \\ 2 \end{pmatrix}.$$

Problème 2

$$[C]_{i,j} = \begin{cases} \sqrt{1/N} \cos \dfrac{(2j+1)i\pi}{2N} & i=0,\ j=0,\dots,N-1 \\ \sqrt{2/N} \cos \dfrac{(2j+1)i\pi}{2N} & i=1,\dots N-1 \\ & j=0,\dots N-1 \end{cases}$$

a) Calculons la matrice de la transformation 2×2.

$$\sqrt{\frac{1}{2}} \cos\left(\frac{2j+1)i\pi}{4}\right. \quad i=0 \text{ et } j=0,1$$

$$\sqrt{\frac{2}{2}} \cos\left(\frac{2j+1)i\pi}{4}\right. \quad i=1, \quad j=0,1$$

$$C_{00} = \sqrt{\frac{1}{2}} \cos(0) = \sqrt{1/2}$$

$$C_{01} = \sqrt{1/2} \cos(0) = \sqrt{1/2}$$

$$C_{10} = \cos\frac{\pi}{4} = \sqrt{2}$$

$$C_{11} = \cos\left(\frac{3\pi}{4}\right) = -\sqrt{2}$$

$$C = \begin{pmatrix} \sqrt{1/2} & \sqrt{1/2} \\ \sqrt{2} & -\sqrt{2} \end{pmatrix} \Rightarrow \begin{matrix}\text{Transformation} \\ C.x\end{matrix}$$