An annotated bibliography of GRASP – Part I: Algorithms

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Abstract

A greedy randomized adaptive search procedure (GRASP) is a metaheuristic for combinatorial optimization. It is a multi-start or iterative process, in which each GRASP iteration consists of two phases, a construction phase, in which a feasible solution is produced, and a local search phase, in which a local optimum in the neighborhood of the constructed solution is sought. Since 1989, numerous papers on the basic aspects of GRASP, as well as enhancements to the basic metaheuristic have appeared in the literature. GRASP has been applied to a wide range of combinatorial optimization problems, ranging from scheduling and routing to drawing and turbine balancing. This is the first of two papers with an annotated bibliography of the GRASP literature from 1989 to 2008. This paper covers algorithmic aspects of GRASP.

Keywords: GRASP; metaheuristics; heuristics; algorithms

1. Introduction

Optimization problems that involve a large finite number of alternatives often arise in the private and public sectors of the economy. In these problems, given a finite solution set X and a real-valued function $f : X \to \mathbb{R}$, one seeks a solution $x^* \in X$ with $f(x^*) \leqslant f(x)$, $\forall x \in X$. Common examples include designing efficient telecommunication networks, constructing cost effective airline crew schedules, and producing efficient routes for waste management pickup. To find the optimal solution in a combinatorial optimization problem it is theoretically possible to enumerate the solutions and evaluate each with respect to the stated objective. However, in practice, it is often infeasible to follow such a strategy of complete enumeration because the number of combinations often grows exponentially with the size of problem.

Much work has been done over the last six decades to develop optimal seeking methods that do not explicitly require an examination of each alternative. This research has given rise to the field of *combinatorial optimization* (see Papadimitriou and Steiglitz, 1982), and an increasing capability to solve ever larger real-world problems. Nevertheless, most problems found in industry and government are either computationally intractable by their nature, or sufficiently large so as to preclude the use of exact algorithms. In such cases, heuristic methods are usually employed to find good, but not necessarily guaranteed optimal, solutions. The effectiveness of these methods depends upon their ability to adapt to a particular realization, avoid entrapment at local optima, and exploit the basic structure of the problem. Building on these notions, various heuristic search techniques have been developed that have demonstrably improved our ability to obtain good solutions to difficult combinatorial optimization problems. The most promising of such techniques include simulated annealing (Kirkpatrick, 1984), tabu search (Glover, 1989, 1990; Glover and Laguna, 1997), genetic algorithms (Goldberg, 1989), variable neighborhood search (Hansen and Mladenović, 1998), and GRASP, or Greedy Randomized Adaptive Search Procedures (Feo and Resende, 1989, 1995).

A GRASP is a multi-start or iterative process (Lin and Kernighan, 1973), in which each GRASP iteration consists of two phases, a construction phase, in which a feasible solution is produced, and a local search phase, in which a local optimum in the neighborhood of the constructed solution is sought. The best overall solution is kept as the result.

In the construction phase, a feasible solution is iteratively constructed, one element at a time. The basic GRASP construction phase is similar to the semi-greedy heuristic proposed independently by Hart and Shogan (1987). At each construction iteration, the choice of the next element to be added is determined by ordering all candidate elements (i.e. those that can be added to the solution) in a candidate list C with respect to a greedy function $g : C \rightarrow \mathbb{R}$. This function measures the (myopic) benefit of selecting each element. The heuristic is adaptive because the benefits associated with every element are updated at each iteration of the construction phase to reflect the changes brought on by the selection of the previous element. The probabilistic component of a GRASP is characterized by randomly choosing one of the best candidates in the list, but not necessarily the top candidate. The list of best candidates is called the *restricted candidate list* (RCL). This choice technique allows for different solutions to be obtained at each GRASP iteration, but does not necessarily compromise the power of the adaptive greedy component of the method.

As is the case for many deterministic methods, the solutions generated by a GRASP construction are not guaranteed to be locally optimal with respect to simple neighborhood definitions. Hence, it is almost always beneficial to apply a local search to attempt to improve each constructed solution. A local search algorithm works in an iterative fashion by successively replacing the current solution by a better solution in the neighborhood of the current solution. It terminates when no better solution is found in the neighborhood. The *neighborhood structure N* for a problem P relates a solution s of the problem to a subset of solutions $N(s)$. A solution s is said to be *locally optimal* if there is no better solution in $N(s)$. The key to success for a local search algorithm consists of the suitable choice of a neighborhood structure, efficient neighborhood search techniques, and the starting solution.

While such local optimization procedures can require exponential time (Johnson et al., 1988) from an arbitrary starting point, empirically their efficiency significantly improves as the initial solution improves. The result is that often many GRASP solutions are generated in the same

amount of time required for the local optimization procedure to converge from a single random start. Furthermore, the best of these GRASP solutions is generally significantly better than the single solution obtained from a random starting point.

It is difficult to formally analyze the quality of solution values found by using the GRASP methodology. However, there is an intuitive justification that views GRASP as a repetitive sampling technique. Each GRASP iteration produces a sample solution from an unknown distribution of all obtainable results. The mean and variance of the distribution are functions of the restrictive nature of the candidate list. For example, if the cardinality of the restricted candidate list is limited to one, then only one solution will be produced and the variance of the distribution will be zero. Given an effective greedy function, the mean solution value in this case should be good, but probably suboptimal. If a less restrictive cardinality limit is imposed, many different solutions will be produced implying a larger variance. Because the greedy function is more compromised in this case, the mean solution value should degrade. Intuitively, however, by order statistics and the fact that the samples are randomly produced, the best value found should outperform the mean value. Indeed, often the best solutions sampled are optimal.

An especially appealing characteristic of GRASP is the ease with which it can be implemented. Few parameters need to be set and tuned, and therefore development can focus on implementing efficient data structures to assure quick GRASP iterations. Finally, GRASP can be trivially implemented in parallel. Each processor can be initialized with its own copy of the procedure, the instance data, and an independent random number sequence. The GRASP iterations are then performed in parallel with only a single global variable required to store the best solution found over all processors.

This paper is the first of two. In both papers, we provide annotated bibliographies of the GRASP literature up to mid-2008. The papers contain references related to GRASP that have either appeared in the literature or as theses. In this paper, we first look at tutorials and surveys. Papers that propose enhancements to the basic heuristic are considered next. Following that, we examine GRASP as a component of a hybrid metaheuristic. Parallelization of GRASP and GRASP source code conclude the paper. In the companion paper (Festa and Resende, 2009), we review the literature of operations research and computer science applications of GRASP as well as industrial applications.

Papadimitriou, C.H., Steiglitz, K., 1982. *Combinatorial optimization: Algorithms and complexity.* Prentice-Hall, Englewood Cliffs, NJ.
A classic book on combinatorial optimization.

Kirkpatrick, S., 1984. Optimization by simulated annealing: quantitative studies. *Journal of Statistical Physics* 34, 975–986.
Description of the simulated annealing metaheuristic.

Glover, F., 1989. Tabu search – part I. *ORSA Journal on Computing* 1, 190–206.
Description of the tabu search metaheuristic.

Glover, F., 1990. Tabu search – part II. *ORSA Journal on Computing* 2, 4–32.
Description of the tabu search metaheuristic.

Glover, F., Laguna, M., 1997. *Tabu Search*. Kluwer Academic Publishers, Dordrecht, MA.
Book on metaheuristics and in particular tabu search.

Goldberg, D.E., 1989. *Genetic Algorithms in Search, Optimization and Machine Learning*. Addison-Wesley, Reading, MA.
Book on genetic algorithms.

Hansen, P., Mladenović, N., 1998. An introduction to variable neighborhood search. In Voss, S., Martello, S., Osman, I.H., Roucairol, C. (eds) *Meta-heuristics, Advances and Trends in Local Search Paradigms for Optimization*. Kluwer Academic Publishers, Dordrecht, MA, pp. 433–458.
Description of the variable neighborhood search metaheuristic.

Feo, T.A., Resende, M.G.C., 1989. A probabilistic heuristic for a computationally difficult set covering problem. *Operations Research Letters* 8, 67–71.
This is the first paper to explicitly describe a GRASP.

Feo, T.A., Resende, M.G.C., 1995. Greedy randomized adaptive search procedures. *Journal of Global Optimization* 6, 109–133.
An early survey of GRASP. See p. 4.

Lin, S., Kernighan, B.W., 1973. An effective heuristic algorithm for the traveling-salesman problem. *Operations Research* 21, 498–516.
An early random multistart local search technique.

Hart, J.P., Shogan, A.W., 1987. Semi-greedy heuristics: an empirical study. *Operations Research Letters* 6, 107–114.
This paper presents a randomized greedy heuristic, called semi-greedy heuristic.

Johnson, D.S., Papadimitriou, C.H., Yannakakis, M., 1988. How easy is local search? *Journal of Computer and System Sciences* 17, 79–100.
Study of the computational complexity of local search.

Festa, P., Resende, M.G.C., 2009. An annotated bibliography of GRASP – Part II: applications. *International Transactions in Operational Research* forthcoming.
Companion paper with an annotated bibliography of applications of GRASP.

2. Tutorials and surveys

Several tutorials and surveys have been published since 1995. Below are a few examples of introductory material on GRASP.

Feo, T.A., Resende, M.G.C., 1995. Greedy randomized adaptive search procedures. *Journal of Global Optimization* 6, 109–133.

In this tutorial paper, the authors define the various components comprising a GRASP. A straightforward implementation of GRASP on a parallel computer is also discussed. The GRASP literature until 1994 is surveyed.

González, J.L., 1996. GRASP. In Díaz, A. (ed.) *Heuristic Optimization and Neural Networks in Operations Management and Engineering*. Editorial Paraninfo, Madrid, pp. 143–161.
This is a chapter on GRASP in a book on heuristic procedures for optimization. In Spanish.

Resende, M.G.C., 2001. Greedy randomized adaptive search procedures (GRASP). In Floudas, C.A., Pardalos, P.M. (eds) *Encyclopedia of Optimization*, Vol. 2. Kluwer Academic Publishers, Dordrecht, MA, pp. 373–382.
A survey of greedy randomized adaptive search procedures. A basic GRASP is explained in detail and enhancements to the procedure are described. Several applications of GRASP are reported, showing how this method can find good approximate solutions to operations research problems and industrial applications.

Ribeiro, C.C., 2002. GRASP: Une métaheuristique gloutone et probabiliste. In Teghem, J., Pirlot, M. (eds) *Optimisation approchée en recherche opérationnelle*, Hermès, pp. 153–176.
A tutorial of GRASP in French.

Pitsoulis, L.S., Resende, M.G.C., 2002. Greedy randomized adaptive search procedures. In Pardalos, P.M., Resende, M.G.C. (eds) *Handbook of Applied Optimization*. Oxford University Press, Oxford, pp. 168–183.
This chapter surveys GRASP. Multi-start heuristics are seen as a way to apply local search to solve combinatorial optimization problems. GRASP is shown to, in some ways, improve upon greedy or random multi-start procedures. Enhancements to GRASP, such as reactive GRASP, hybrid GRASP, and use of long-term memory are discussed. The parallelization of GRASP is also considered. The chapter ends with a survey of GRASP for solving problems in logic, assignment, and location.

Festa, P., Resende, M.G.C., 2002. GRASP: an annotated bibliography. In Ribeiro, C.C., Hansen, P. (eds) *Essays and Surveys on Metaheuristics*. Kluwer Academic Publishers, Dordrecht, MA, pp. 325–367.
Since 1989, numerous papers on the basic aspects of GRASP, as well as enhancements to the basic metaheuristic have appeared in the literature. GRASP has been applied to a wide range of combinatorial optimization problems, ranging from scheduling and routing to drawing and turbine balancing. This paper is an annotated bibliography of the GRASP literature from 1989 to 2001.

Martí, R., 2003. Multi-start methods. In Glover, F., Kochenberger, G. (eds) *Handbook of Metaheuristics*. Kluwer Academic Publishers, Dordrecht, MA.
Heuristic search procedures usually require some type of diversification to overcome local optimality. In this chapter the author describes a classification of these methods in terms of their use of randomization, memory, and degree of rebuild. A computational comparison is described, showing different methods on a linear ordering problem.

Resende, M.G.C., Ribeiro, C.C., 2003. Greedy randomized adaptive search procedures. In Glover, F., Kochenberger, G. (eds) *Handbook of Metaheuristics*. Kluwer Academic Publishers, Dordrecht, MA, pp. 219–249.
In this chapter, the authors describe the basic components of GRASP as well as successful implementation techniques and parameter tuning strategies. Enhanced or alternative solution construction mechanisms and techniques to speed up the search are also described. These include reactive GRASP, cost perturbations, bias functions, memory and learning, local search on partially constructed solutions, hashing, and filtering. The authors also discuss implementation strategies of memory-based intensification and post-optimization techniques using path-relinking, hybridizations with other metaheuristics, and parallelization strategies. Applications are also reviewed.

Resende, M.G.C., Ribeiro, C.C., 2005a. Parallel greedy randomized adaptive search procedures. In Alba, E. (ed.) *Parallel Metaheuristics: A new class of algorithms*. John Wiley and Sons, New York, pp. 315–346.
In this chapter, the authors survey parallel implementations of GRASP. They describe simple strategies to implement independent parallel GRASP heuristics and more complex cooperative schemes using a pool of elite solutions to intensify the search process. Some applications of independent and cooperative parallelizations are presented.

Resende, M.G.C., Ribeiro, C.C., 2005b. GRASP with path-relinking: recent advances and applications. In Ibaraki, T., Nonobe, K., Yagiura, M. (eds) *Metaheuristics: Progress as Real Problem Solvers*. Springer, Berlin, pp. 29–63.
Several recent applications of GRASP with path relinking are reviewed. Path-relinking adds a memory mechanism to GRASP by providing an intensification strategy that explores trajectories connecting GRASP solutions and elite solutions previously produced during the search. This paper reviews recent advances and applications of GRASP with path-relinking and discusses extensions of this strategy. In particular, parallel implementations and applications of path relinking with other metaheuristics are addressed.

Resende, M.G.C., González Velarde, J.L., 2003. GRASP: Procedimientos de busqueda miope aleatorizado y adaptatitvo. *Inteligencia Artificial* 19, 61–76.
Here, the authors describe the basic components of GRASP as well as successful implementation techniques and parameter tuning strategies. In Spanish.

Resende, M.G.C., Ribeiro, C.C., 2008. Greedy randomized adaptive search procedures: advances and applications. In Gendreau, M., Potvin, J.Y. (eds) *Handbook of Metaheuristics* (2nd edn.). Springer, Berlin.
This chapter gives an overview of GRASP. Besides describing the basic building blocks of a GRASP, it covers enhancements to the basic procedure, including reactive GRASP, hybrid GRASP, and intensification strategies. Hybridizations with other metaheuristics are also reviewed. To appear.

Resende, M.G.C., 2008. Metaheuristic hybridization with GRASP. In Chen, Z.-L., Raghavan, S. (eds) *TutORials in Operations Research*. Institute for Management Science and Operational Research.

This tutorial describes different ways to hybridize GRASP and create new and more effective metaheuristics. To appear.

3. Enhancements to the basic GRASP

The standard (or basic) GRASP consists of repeated applications of GRASP construction (using a fixed candidate list strategy), followed by hill climbing local search. Enhancements to this basic GRASP are found in the following papers.

Bresina, J.L., 1996. Heuristic-biased stochastic sampling. Proceedings of the AAAI-96, pp. 271–278.
This paper presents a generalization of iterative sampling called Heuristic-Biased Stochastic Sampling (HBSS). The two search techniques have the same overall control structure. The difference lies in how a choice is made at each decision point. HBSS uses a given search heuristic to focus its exploration. The degree of focusing is determined by a *bias function* that reflects the confidence one has in the heuristic's accuracy. This methodology can be directly applied in a GRASP construction phase, by biasing the selection of RCL elements to favor those with higher greedy function values.

Mockus, J., Eddy, E., Mockus, A., Mockus, L., Reklaitis, G.V., 1997. *Bayesian Discrete and Global Optimization.* Kluwer Academic Publishers, Dordrecht, MA.
This book describes the Bayesian approach to discrete optimization. A Bayesian heuristic algorithm version of GRASP is described.

Atkinson, J.B., 1998. A greedy randomised search heuristic for time-constrained vehicle scheduling and the incorporation of a learning strategy. *Journal of the Operational Research Society* 49, 700–708.
Two forms of adaptive search called *local* and *global* adaptation are identified. In both search techniques, the greedy function takes into account a quantity that measures heuristically the quality of the partial solution. While in local adaptation the decisions made within a particular run influence only the subsequent performance of the heuristic, global adaptation involves making decisions that affect the performance of the heuristic in subsequent runs.

Fleurent, C., Glover, F., 1999. Improved constructive multistart strategies for the quadratic assignment problem using adaptive memory. *INFORMS Journal on Computing* 11, 198–204.
A study of alternatives to memoryless multistart approaches like GRASP is conducted for the quadratic assignment problem. Adaptive memory strategies retain and analyze features of selected solutions in order to provide a basis for improving future executions of the construction procedure.

Laguna, M., Martí, R., 1999. GRASP and path relinking for 2-layer straight line crossing minimization. *INFORMS Journal on Computing* 11, 44–52.
This paper introduces GRASP with path relinking, which incorporates to GRASP a path relinking strategy to search for improved outcomes. Path relinking generates new solutions by exploring trajectories connecting high quality solutions. Starting from an *initiating solution*, path

relinking generates a path in the neighborhood space that leads toward the other solutions, called *guiding solutions*. This is accomplished by selecting moves that introduce attributes contained in the guiding solutions.

Prais, M., 2000. Parameter variation in GRASP procedures. Ph.D. thesis, Department of Computer Sciences, Catholic University of Rio de Janeiro, Rio de Janeiro, Brazil.
This thesis describes a GRASP for a matrix decomposition problem in TDMA traffic assignment. It proposes Reactive GRASP, a refinement of GRASP where the RCL parameter is adjusted dynamically to favor values that produce good solutions. Reactive GRASP is compared with other RCL strategies on matrix decomposition, set covering, maximum satisfiability, and graph planarization.

Prais, M., Ribeiro, C.C., 2000a. Reactive GRASP: an application to a matrix decomposition problem in TDMA traffic assignment. *INFORMS Journal on Computing* 12, 164–176.
A refinement of GRASP, called Reactive GRASP, is proposed. Instead of using a fixed value for the basic parameter that defines the restrictiveness of the candidate list during the construction phase, Reactive GRASP self-adjusts the parameter value according to the quality of the solutions previously found.

Prais, M., Ribeiro, C.C., 2000b. Parameter variation in GRASP procedures. *Investigación Operativa* 9, 1–20.
The GRASP RCL parameter α that determines the size of the restricted candidate list can be adjusted, leading to different behavior of the GRASP implementation. This paper investigates four strategies for the variation of the parameter α: (1) reactive – α is self-adjusted along the iterations; (2) uniform – α is randomly chosen from a discrete uniform probability distribution; (3) hybrid – α is randomly chosen from a fixed probability distribution concentrated around the value corresponding to the purely greedy choice; (4) fixed – α has a fixed value close to the purely greedy choice. The reactive strategy is the most flexible and adherent to the characteristics of the specific problem to be solved. In Portuguese.

Binato, S., Faria, H. Jr., Resende, M.G.C., 2001. Greedy randomized adaptive path relinking. Proceedings of the 4th Metaheuristics International Conference (MIC2001), pp. 393–397.
This paper proposes a new metaheuristic approach called Greedy Randomized Adaptive Path Relinking (GRAPR). It uses generalized GRASP concepts, such as the RCL mechanism, in path relinking to explore different trajectories between two good-quality solutions previously found.

Ribeiro, C.C., Uchoa, E., Werneck, R.F., 2002. A hybrid GRASP with perturbations for the Steiner problem in graphs. *INFORMS Journal on Computing* 14, 228–246.
This paper describes a hybrid GRASP with weight perturbations and adaptive path-relinking heuristic (HGP-PR) for the Steiner problem in graphs. In this multi-start approach, the greedy randomized construction phase of a GRASP is replaced by the combination of several construction heuristics with a weight perturbation strategy. A strategic oscillation scheme combining intensification and diversification elements is used for the perturbation of the original weights. The improvement phase circularly explores two different local search strategies.

An adaptive path-relinking technique is applied to a set of elite solutions as an intensification strategy.

Hirsch, M.J., Meneses, C.N., Pardalos, P.M., Resende, M.G.C., 2007. Global optimization by continuous GRASP. *Optimization Letters* 1, 2, 201–212.

Continuous GRASP (C-GRASP) extends GRASP from the domain of discrete optimization to that of continuous global optimization. The main difference is that an iteration of C-GRASP does not consist of a single greedy randomized construction followed by local improvement, but rather a series of construction-local improvement cycles with the output of construction serving as the input of the local improvement, as in GRASP, and the output of the local improvement serving as the input of the construction procedure, unlike GRASP. C-GRASP works by discretizing the domain into a uniform grid. Both the construction and local improvement phases move along points on the grid. As the algorithm progresses, the grid adaptively becomes more dense.

Andrade, D.V., Resende, M.G.C., June 2007. GRASP with evolutionary path-relinking. Proceedings of The Seventh Metaheuristics International Conference (MIC2007).

To solve a network migration problem the authors propose GRASP with evolutionary path-relinking (EvPR), a metaheuristic resulting from the hybridization of GRASP, path-relinking, and evolutionary path-relinking. The solutions in the pool are evolved as a series of populations P_1, $P_2, \ldots$ of equal size. The initial population is the pool of elite solutions produced by GRASP with PR. In iteration k of EvPR, path-relinking is applied between a set of pairs of solutions in population P_k and, with the same rules used to test for membership in the pool of elite solutions, each resulting solution is tested for membership in population P_{k+1}. This evolutionary process is repeated until no improvement results from one population to the next.

Hirsch, M.J., Pardalos, P.M., Resende, M.G.C., 2008. Speeding up continuous GRASP. *European Journal of Operational Research.*

The authors propose some improvements to speed up the original continuous GRASP (C-GRASP) and to make it more robust. The authors compare the new C-GRASP with the original version as well as with other algorithms from the literature on a set of benchmark multimodal test functions whose global minima are known. To appear.

4. GRASP in hybrid metaheuristics

GRASP has been used in conjunction with other metaheuristics. The following papers illustrate these hybrid techniques.

Laguna, M., González-Velarde, J.L., 1991. A search heuristic for just-in-time scheduling in parallel machines. *Journal of Intelligent Manufacturing* 2, 253–260.

The proposed hybrid metaheuristic combines elements of GRASP with elements of tabu search.

Yagiura, M., Ibakari, T., 1996. Genetic and local search algorithms as robust and simple optimization tools. In *Meta-heuristics: Theory and Applications.* Kluwer Academic Publishers, Dordrecht, MA, pp. 63–82.

Various metaheuristics, such as random multi-start local search (MLS) and genetic algorithm (GA), are implemented and their performance compared. The objective of the authors is not to propose the most powerful technique, but rather to compare general tendencies of various algorithms. From their analysis it results that a GRASP type modification of MLS improves its performance.

Delmaire, H., Díaz, J.A., Fernández, E., Ortega, M., 1999. Reactive GRASP and tabu search based heuristics for the single source capacitated plant location problem. *INFOR* 37, 194–225.
A hybrid heuristic is proposed. It embeds a reactive GRASP in a tabu search algorithm as a diversification method that provides elite candidate sets.

Ahuja, R.K., Orlin, J.B., Tiwari, A., 2000. A greedy genetic algorithm for the quadratic assignment problem. *Computers and Operations Research* 27, 917–934.
A hybrid genetic algorithm for the QAP is presented. GRASP is used to generate the initial population.

Armony, M., Klincewicz, J.G., Luss, H., Rosenwein, M.B., 2000. Design of stacked self-healing rings using a genetic algorithm. *Journal of Heuristics* 6, 85–105.
A genetic algorithm for design of stacked self-healing rings is proposed. The objective is to optimize the trade-off between the cost of connecting nodes to the ring and the cost of routing demand on multiple rings. The initial population of the genetic algorithm is made up of randomly generated solutions as well as solutions generated by a GRASP. Computational comparisons are made with a commercial integer programming package.

Abdinnour-Helm, S., Hadley, S.W., 2000. Tabu search based heuristics for multi-floor facility layout. *International Journal of Production Research* 38, 365–383.
Two 2-stage heuristics are proposed for solving the multi-floor facility layout problem. GRASP/TS applies a GRASP to find the initial layout and tabu search to refine the initial layout. Computational tests indicate that GRASP/TS compares favorably with heuristics that do not rely on exact algorithms.

Lourenço, H.R., Paixão, J.P., Portugal, R., 2001. Multiobjective metaheuristics for the bus-driver scheduling problem. *Transportation Science* 35, 331–343.
GRASP is used in a genetic algorithm to implement a type of crossover called *perfect offspring*.

Colomé, R., Serra, D., 2001. Consumer choice in competitive location models: Formulations and heuristics. *Papers in Regional Science* 80, 439–464.
A hybrid GRASP that uses tabu search in the local search phase is proposed.

Festa, P., Pardalos, P.M., Resende, M.G.C., Ribeiro, 2001. GRASP and VNS for MAX-CUT. In Sousa, J.P. (ed.) Proceedings of the IV Metaheuristics International Conference (MIC2001), pp. 371–376.
A GRASP and a variable neighborhood search (VNS) for MAX-CUT are proposed and tested. In the case of the MAX-CUT problem, it is intuitive to relate the greedy function

to the sum of the weights of the edges in each cut. Local search is based on Boolean flip operations.

Ochi, L.S., Silva, M.B., Drummond, L., 2001. Distributed parallel metaheuristics based on GRASP and VNS for solving the traveling purchaser problem. Proceedings of the 4th Metaheuristics International Conference (MIC2001), pp. 489–494.
Hybrid metaheuristics, based on GRASP and VNS, are described to solve a generalization of the traveling salesman problem, called the traveling purchaser problem.

Cano, J.R., Cordón, O., Herrera, F., Sánchez, L., 2002. A greedy randomized adaptive search procedure for the clustering problem. *International Journal of Intelligent and Fuzzy Systems* 12, 235–242.
A GRASP for cluster analysis is described. The best solutions are found with a hybrid GRASP/*K*-means with Kaufman initialization.

Talbi, E.-G., 2002. A taxonomy of hybrid metaheuristics. *Journal of Heuristics* 8, 541–564.
A taxonomy of hybrid metaheuristics is presented in an attempt to provide a common terminology and some classification mechanisms.

Drummond, L., Vianna, L.S., Silva, M.B., Ochi, L.S., 2002. Distributed parallel metaheuristics based on GRASP and VNS for solving the traveling purchaser problem. Proceedings of the Ninth International Conference on Parallel and Distributed Systems (ICPADS02), pp. 257–266.
Several strategies for parallel implementations of GRASP and VNS applied to the traveling purchaser problem are described. Parallel algorithms based on master-worker, completely distributed and independent models, using static and dynamic load balance are proposed.

Festa, P., Pardalos, P.M., Resende, M.G.C., Ribeiro, C.C., 2002. Randomized heuristics for the MAX-CUT problem. *Optimization Methods and Software* 7, 1033–1058.
A GRASP, a variable neighborhood search (VNS), and a path-relinking heuristic for MAX-CUT are proposed and tested. New hybrid heuristics that combine GRASP, VNS as GRASP local search, and path-relinking are also proposed and tested. The greedy function is related to the sum of the weights of the edges in each cut. Local search is based on Boolean flip operations. On a set of standard test problems, new best-known solutions are produced for several of the instances.

Ribeiro, C.C., Uchoa, E., Werneck, R.F., 2002. A hybrid GRASP with perturbations for the Steiner problem in graphs. *INFORMS Journal on Computing* 14, 228–246.
A hybrid GRASP with weight perturbations and adaptive path-relinking heuristic (HGP-PR) is proposed for the Steiner problem in graphs. See p. 9.

Lourenço, H.R., Serra, D., 2002. Adaptive approach heuristics for the generalized assignment problem. *Mathware and Soft Computing* 90, 2–3, 209–234.
An ant colony optimization algorithm (MAX-MIN Ant System) and a GRASP are embedded in a general framework. The main difference between GRASP and the ant colony optimization algorithm is the method of selecting the agent to whom the previously chosen task is assigned. For

GRASP the choice is a probabilistic bias according to an adaptive priority function that does not depend on the solutions seen in previous iterations of the general framework but only on the choices done in the previous iterations. The local search procedure interchanges or reassigns tasks. Infeasible solutions with respect to capacity constraints are admitted.

Andreatta, A.A., Ribeiro, C.C., 2002. Heuristics for the phylogeny problem. *Journal of Heuristics* 8, 429–447.

Different heuristic approaches are proposed for the phylogeny problem under the parsimony criterion, including a GRASP and a VNS. The greedy randomized construction randomly selects a taxon-branch pair from among all those leading to the most parsimonious increment value. The local search phase is implemented with the selection of the first improving move. Two neighborhood strategies are used to devise two alternative GRASP algorithms. The first one applies subtree pruning and regrafting (SPR), while the second one defines three different neighborhoods explored within a VND procedure: SPR, a nearest neighborhood interchange (NNI), and the single step neighborhood (STEP), where a neighbor is obtained by taking out a taxon (i.e. a leaf) from the current solution and putting it back into another branch of the tree.

Martí, R., Laguna, M., 2003. Heuristics and meta-heuristics for 2-layer straight line crossing minimization. *Discrete Applied Mathematics* 1270, 3, 665–678.

A tabu search that yields the best results for relatively dense graphs and a GRASP that outperforms other approaches on low-density graphs are proposed. The GRASP construction phase starts by creating a list of unassigned vertices, originally consisting of all the vertices in the graph. The greedy criterion takes into account vertex degree. At each iteration of the local search phase vertices are selected to be moved and the probability of selecting a vertex increases with its degree. When a vertex is selected, three moves are considered.

Resende, M.G.C., Ribeiro, C.C., 2003. A GRASP with path-relinking for private virtual circuit routing. *Networks* 41, 104–114.

Variants of a GRASP with path relinking for private virtual circuit routing are proposed. In the construction phase, the routes are determined one at a time. At each iteration, an RCL is formed by a certain fixed number of unrouted PVC pairs with the largest demands. From the RCL a pair is then selected with a probability computed through a strategy usually employed by heuristic-biased stochastic sampling. Once a PVC is selected, it is routed on a shortest path from its origin to its destination. Each solution built in the construction phase may be viewed as a set of routes, one for each PVC. The local search procedure seeks to improve each route in the current solution, removing some units of flow from each edge in its current route and trying to route.

Delorme, X., Gandibleux, X., Rodriguez, J., 2004. GRASP for set packing problems. *European Journal of Operational Research* 1530, 3, 564–580.

A GRASP is described for the set packing problem. The proposed framework uses a self-tuning procedure (reactive GRASP), an intensification procedure (using path relinking), and a procedure involving the diversification of the selection (using a learning process). A set of test problem instances includes real-world railway planning instances never solved before by means of metaheuristics.

Cotta, C., Fernández, A.J., 2004. A hybrid GRASP with evolutionary algorithm approach to Golomb ruler search. In *Parallel Problem Solving from Nature – PPSN VIII*, Vol. 3242 of *Lecture Notes in Computer Science*. Springer-Verlag, Berlin, pp. 481–490.
A GRASP for Golomb Ruler search is proposed. It takes as input the conflict graph associated with the problem. The constructive phase greedy criterion is based on vertex degrees, because the degree of a vertex is a measure of labels in conflict. To build the RCL a fixed-size criterion is adopted. Starting from the built solution, the local search procedure tests for each point another valid candidate position and performs the best change. Computational results show that GRASP generates better solutions than all those reported in the literature in reasonable computational times.

Rocha, P.L., Ravetti, M.G., Mateus, G.R., 2004. The metaheuristic GRASP as an upper bound for a branch and bound algorithm in a scheduling problem with non-related parallel machines and sequence-dependent setup times. In *Proceedings of the 4th EU/ME Workshop: Design and Evaluation of Advanced Hybrid Meta-Heuristics*. Vol. 1. pp. 62–67.
A hybridization of GRASP as an upper bound for a branch and bound procedure is proposed. At each GRASP iteration, the greedy criterion consists in sorting the jobs in a non-decreasing order of due date and then assigning each job to the machine able to finishing it first. The local search switches all existing pairs of jobs assigned to different machines. The authors have also implemented path relinking (PR) at the end of each GRASP iteration to intensify the local search.

Piñana, E., Plana, I., Campos, V., Martí. R., 2004. GRASP and path relinking for the matrix bandwidth minimization. *European Journal of Operational Research* 1530, 1, 200–210.
Five GRASP construction methods are proposed and tested. The first method, C_1, starts by selecting at random a vertex and assigning to it a random label. Then, at each iteration the candidate elements CL are all vertices that are adjacent to at least one labeled vertex and the RCL is formed by those candidate vertices with a maximum number of adjacent labeled vertices. Constructive methods C_2 and C_3 differ from C_1 in the definition of the RCL. At each iteration, in C_2 RCL elements are those vertices that have been in CL for a maximum number of construction steps, while C_3 combines both criteria by considering the attractiveness of a vertex as the sum of both measures: the number of adjacent labeled vertices and the number of steps that a vertex has been in CL. Constructive methods C_4 and C_5 are partially based on the construction of a level structure of vertices set proposed in the GPS method, i.e. a special partition of the set of vertices. The local search considers the changes in the number of critical vertices before and after a move of a vertex.

Souza, M.C., Duhamel, C., Ribeiro, C.C., 2004. A GRASP heuristic for the capacitated minimum spanning tree problem using a memory-based local search strategy. In Resende, M.G.C., Sousa, J. (eds) *Metaheuristics: Computer Decision-Making*. Kluwer Academic Publisher, Dordrecht, MA, pp. 627–658.
A GRASP with path-relinking heuristic for the capacitated minimum spanning tree problem is proposed. It uses a randomized version of a savings heuristic in the construction phase and an extension of the local search strategy, incorporating some short-term memory elements of tabu search. The greedy criterion takes into account the savings with respect to edge costs

and capacity constraints. The local search strategy is based on a new neighborhood structure defined by path exchanges.

Commander, C.W., Butenko, S.I., Pardalos, P.M., Oliveira, C.A.S., 2004. Reactive GRASP with path relinking for the broadcast scheduling problem. Proceedings of the 40th Annual International Telemetry Conference, pp. 792–800.
Several variations of GRASP with path relinking for the broadcast scheduling problem are proposed. A reactivity method to balance GRASP parameters is also described. The greedy choice consists in sorting the stations in descending order of the number of one-hop and two-hop neighbors. The local search procedure sorts the slots in descending order of the number of bursts. The two slots with the fewest transmissions are combined. A colliding station from the combined slot is then randomly selected and every attempt is made to swap this station with another from the remaining slots.

Oliveira, C.A., Pardalos, P.M., Resende, M.G.C., 2004. GRASP with path-relinking for the quadratic assignment problem. In *Efficient and Experimental Algorithms*, Vol. 3059 of *Lecture Notes in Computer Science*, pp. 356–368.
A GRASP for the quadratic assignment problem is described. Construction first makes two assignments, and then completes the solution by making assignments, one at a time. The greedy function is assignment interaction cost. The local search procedure is a two-assignment exchange. Path-relinking is invoked at each GRASP iteration as an intensification procedure.

Lim, A., Wang, F., 2004. A smoothed dynamic tabu search embedded GRASP for m-VRPTW. Proceedings of ICTAI 2004, pp. 704–708.
m-VRPTW is a vehicle routing problem with both time window and limited number of vehicles (m-VRPTW). A GRASP is proposed that uses techniques including multiple initialization and solution reuse. A smoothed dynamic tabu search is also embedded into the GRASP to improve the performance.

Beltrán, J.D., Calderón, J.E., Cabrera, R.J., Pérez, J.A.M., Moreno-Vega, J.M., 2004. GRASP/VNS hybrid for the strip packing problem. Proceedings of Hybrid Metaheuristics (HM2004), pp. 79–90.
The authors propose a hybrid metaheuristic that combines GRASP and VNS procedure as a local search.

de la Pena, M.G.B., 2004. Heuristics and metaheuristics approaches used to solve the rural postman problem: a comparative case study. Proceedings of the Fourth International ICSC Symposium on Engineering of Intelligent Systems (EIS 2004).
A new hybrid approach for the rural postman problem, based on simulated annealing, GRASP, and genetic algorithm, is proposed.

Li, Z., Guo, S., Wang, F., Lim, A., 2004. Improved GRASP with tabu search for vehicle routing with both time window and limited number of vehicles. In Orchard, B., Yang, C., Ali, M. (eds) *Innovations in Applied Artificial Intelligence – Proceedings of the 17th International Conference on Industrial and Engineering Applications of Artificial Intelligence and Expert Systems*

(IEA/AIE 2004), Vol. 3029 of *Lecture Notes in Computer Science*. Springer-Verlag, Berlin, pp. 552–561.
The paper proposes an improved GRASP by techniques of multiple initializations, solution reuse, and mutation improvement, with four state-of-the-art heuristics: short left time first, near customer first, short waiting time first, and long route first.

Pérez, M., Almeida, F., Moreno-Vega, J.M., 2005. A hybrid GRASP-path relinking algorithm for the capacitated hub median problem. In *Hybrid Metaheuristics*, Vol. 3636 of *Lecture Notes in Computer Science*, Springer, Berlin, pp. 142–153.
The greedy evaluation function has two phases: a location phase and an allocation phase. In the location phase one element is added at a time to the set of hubs, while the allocation phase consists of an allocation to the nearest hub. As local search, the authors use a greedy procedure for location and an allocation to the nearest hub.

Cebrián, M., Dotú, I., 2005. A simple hybrid GRASP-evolutionary algorithm for CSPs. Proceedings of the 2nd International Workshop on Local Search Techniques in Constraint Satisfaction (LSCS05), pp. 2–16.
A hybrid heuristic is proposed featuring a GRASP-like mechanism for genotype-to-phenotype mapping. The problem consists in searching for an optimal variable ordering. The algorithm maintains a population of GRASP parameters and performs a number of iterations until a solution is found. Each iteration selects two individuals in the population and crosses and/or mutates them. The next population is obtained in an elitist fashion.

Santos, L.F., Ribeiro, M.H., Plastino, A., Martins, S.L., 2005. A hybrid GRASP with data mining for the maximum diversity problem. In *Hybrid Metaheuristics*, Vol. 3636 of *Lecture Notes in Computer Science*. Springer, Berlin, pp. 116–127.
The authors describe how to combine GRASP with data mining techniques to solve the maximum diversity problem. Data mining refers to the extraction of new and potentially useful knowledge from datasets in terms of patterns and rules.

Faria, H. Jr., Binato, S., Resende, M.G.C., Falcão, D.J., 2005. Power transmission network design by a greedy randomized adaptive path relinking approach. *IEEE Transactions on Power Systems* 200, 1, 43–49.
This paper proposes a new metaheuristic approach called Greedy Randomized Adaptive Path-Relinking (GRAPR), to solve the static power transmission network design problem. GRAPR uses generalized GRASP concepts, such as the RCL concept, in the path-relinking phase to explore different trajectories between two good quality solutions previously found. Computational results obtained from two real-world case studies of Brazilian systems are described.

Ribeiro, C.C., Vianna, D.S., 2005. A GRASP/VND heuristic for the phylogeny problem using a new neighborhood structure. *International Transactions in Operational Research* 12, 325–338.
A phylogeny is a tree that relates taxonomic units, based on their similarity over a set of characters. The phylogeny problem consists in finding a phylogeny with the minimum number of

evolutionary steps. The authors propose a GRASP that uses variable neighborhood descent for local search. At a generic construction phase iteration, a pair taxon-branch is randomly selected from among all those with cost 10% higher than the most parsimonious increment value. The neighborhood defined is the subtree pruning and regrafting (SPR or 1-SPR): a subtree of the current tree is disconnected and reconnected in a different position.

Festa, P., Pardalos, P.M., Pitsoulis, L.S., Resende, M.G.C., 2005. GRASP with path-relinking for the weighted maximum satisfiability problem. In *Proceedings of the IV Workshop on Efficient and Experimental Algorithms (WEA2005)*, Vol. 3503 of *Lecture Notes in Computer Science*, Springer, Berlin, pp. 367–379.
A GRASP with path-relinking for finding good quality solutions of the weighted maximum satisfiability problem (MAX-SAT) is described. In the construction phase, given any partial solution corresponding to a set of satisfied clauses, the next candidate element added to the solution maximizes the total weight of the unsatisfied clauses that become satisfied after the assignment. Once completed a truth assignment is obtained, local search is applied using the simple-flip neighborhood.

Pacheco, J.A., Casado, S., 2005. Solving two location models with few facilities by using a hybrid heuristic: a real health resources case. *Computers and Operations Research* 32, 3075–3091.
A hybrid scatter search algorithm is proposed that incorporates procedures based on different strategies, such as GRASP and path relinking. GRASP is used as a diversification method and the greedy criterion takes into account the value of the objective function if a certain location is added to the solution. Local search is based on interchange neighborhood.

Festa, P., Pardalos, P.M., Pitsoulis, L.S., Resende, M.G.C., 2006. GRASP with path-relinking for the weighted MAXSAT problem. *ACM Journal of Experimental Algorithmics* 11, 1–16.
A GRASP with path-relinking for finding good quality solutions of the weighted maximum satisfiability problem (MAX-SAT) is described. Construction and local search phase details are as in Festa et al. (2005).

Lim, A., Rodrigues, B., Wang, C., 2006. Two-machine flow shop problems with a single server. *Journal of Scheduling* 9, 515–543.
Several heuristics are proposed to solve a two-machine flow shop-scheduling problem with a single server. They include simulated annealing, tabu search, genetic algorithms, GRASP, and other hybrids. In the construction phase two greedy criteria are adopted. One criterion is to sort by setup times instead of the processing times. The second criterion schedules all jobs one by one and each insertion increases the objective function by a minimum value. For the local search phase, the authors define several neighborhoods, including swapping, insertion, and a mixture of these two.

Raidl, G.R., 2006. A unified view on hybrid metaheuristics. In: Almeida, F., Aguilera, M.J.B., Blum, C., Vega, J.M.M., Pérez, M.P., Roli, A., Sampels, M. (eds) *Hybrid Metaheuristics*, Vol. 4030 of *Lecture Notes in Computer Science*. Springer, Berlin, pp. 1–12.
This paper overviews several popular hybridization approaches. With respect to low-level hybrids of different metaheuristics, a unified view based on a common pool template is described.

Jourdan, L., Dhaenens, C., Talbi, E.-G., October 2006. Using datamining techniques to help metaheuristics: a short survey. In Almeida, F., Aguilera, M.J.B., Blum, C., Vega, J.M.M., Pérez, M.P., Roli, A., Sampels, M. (eds) *Hybrid Metaheuristics, Third International Workshop, HM 2006*, Vol. 4030 of *Lecture Notes in Computer Science*. Springer, Berlin.
The authors present a short survey enumerating opportunities to combine metaheuristics and data mining.

Ribeiro, M.H., Plastino, A., Martins, S.L., 2006. Hybridization of GRASP metaheuristic with data mining techniques. *Journal of Mathematical Modelling and Algorithms* 5, 23–41.
The authors describe how to combine GRASP with data mining techniques.

Santos, L.F., Milagres, R., Albuquerque, C.V., Martins, S., Plastino, A., 2006. A hybrid GRASP with data mining for efficient server replication for reliable multicast. 49th Annual IEEE GLOBECOM Technical Conference.
The authors describe how to combine GRASP with data mining techniques for efficient server replication for reliable multicast. Data mining refers to the extraction of new and potentially useful knowledge from datasets.

Díaz, J.A., Fernández, E., 2006. Hybrid scatter search and path relinking for the capacitated *p*-median problem. *European Journal of Operational Research* 169, 570–585.
GRASP is used to generate the initial reference set both for scatter search and path relinking.

Pu, G.G., Chong, Z., Qiu, Z.Y., Lin, Z.Q., He, J.F., 2006. A hybrid heuristic algorithm for HW-SW partitioning within timed automata. In *Proceedings of Knowledge-based Intelligent Information and Engineering Systems*, Vol. 4251 of *Lecture Notes in Artificial Intelligence*. Springer-Verlag, Berlin, pp. 459–466.
The authors integrate the timed automata model into a hybridization of GRASP and tabu search. A tabu search heuristic is used as the local search procedure.

Dias, T.C.S., de Sousa Filho, G.F., Macambira, E.M., Cabral, L.A.F., Fampa, M.H.C., 2006. An efficient heuristic for the ring star problem. In *Experimental Algorithms*, Vol. 4007 of *Lecture Notes in Computer Science*. Springer, Berlin, pp. 24–35.
A hybrid metaheuristic is proposed for the Ring Star Problem. It uses a General Variable Neighborhood Search (GVNS) to improve the quality of the solution obtained with a GRASP.

Andrade, D.V., Resende, M.G.C., June 2007. GRASP with evolutionary path-relinking. Proceedings of The Seventh Metaheuristics International Conference (MIC2007).
The authors propose GRASP with evolutionary path-relinking (EvPR), a metaheuristic resulting from the hybridization of GRASP, path-relinking, and evolutionary path-relinking.

Ribeiro, C.C., Urrutia, S., 2007. Heuristics for the mirrored traveling tournament problem. *European Journal of Operational Research* 179, 775–787.
The Traveling Tournament Problem models some sport timetabling issues, where the objective is to minimize the total distance traveled by the teams. The authors study the mirrored version

of the problem. They propose a fast constructive algorithm and a new heuristic based on a combination of GRASP and iterated local search. A neighborhood based on ejection chains is also proposed.

Andrade, D.V., Resende, M.G.C., 2007. GRASP with path-relinking for network migration scheduling. Proceedings of the International Network Optimization Conference (INOC 2007). To approximately solve both versions of a network migration problem, the authors propose a hybrid heuristic which combines GRASP with path-relinking. In the construction phase, given an initial partial sequence of vertices, the greedy choice is to augment the sequence by adding a vertex to a position where the increase in the objective function is minimized. A two-swap neighborhood is used in the local search procedure.

Duarte, A., Martí, R., 2007. Tabu search and GRASP for the maximum diversity problem. *European Journal of Operational Research* 1780, 1, 71–84.
The authors propose a new heuristic based on tabu search that incorporates memory structures for both construction and improvement. They compare their tabu search construction with a memoryless design and with previous algorithms recently developed for this problem. The constructive method can be coupled with a local search procedure or a short-term tabu search for improved outcomes.

Boudia, M., Louly, M.A.O., Prins, C., 2007. A reactive GRASP and path relinking for a combined production-distribution problem. *Computers and Operations Research* 34, 3402–3419.
An NP-hard production-distribution problem for one product over a multi-period horizon is studied. The aim is to minimize total cost taking production setups, inventory levels, and distribution. A GRASP and two improved versions using either a reactive mechanism or path-relinking are proposed. The greedy criterion takes into account the best insertion position and the minimum associated cost. Local search considers different types of moves that for each customer changes the quantity delivered, the day of production, the day of delivery, the delivery trip, and the position in this trip.

Fernandes, S., Lourenço, H.R., 2007. A GRASP and branch-and-bound metaheuristic for the job-shop scheduling. In *Evolutionary Computation in Combinatorial Optimization*, Vol. 4446 of *Lecture Notes in Computer Science*. Springer, Berlin, pp. 60–71.
This paper presents a simple algorithm for the job shop scheduling problem that combines the local search heuristic GRASP with a branch-and-bound exact method of integer programming.

Goodman, M.D., Dowsland, K.A., Thompson, J.M., 2007. A GRASP-knapsack hybrid for a nurse-scheduling problem. *Journal of Heuristics*. Published online 27 November 2007. doi:10.1007/s10732-007-9066-7
Instead of selecting the next element to be inserted in the partial solution purely on a myopic basis, the authors employ a look-ahead strategy based on a knapsack model. In the local search, a move consists in changing the shift pattern of a single nurse.

de Lima, F.C., de Melo, J.D., Neto, A.D.D., 2007. Proposal for improvement of GRASP metaheuristic and genetic algorithm using the Q-Learning algorithm. In *Proceedings of the*

Seventh International Conference on Intelligent Systems Design and Applications. IEEE Computer Society, pp. 465–470.
This paper proposes the use of a technique of reinforcement learning called the Q-Learning Algorithm for the construction phase of GRASP and also as generator of the initial population for a genetic algorithm.

Santos, L.F., Martins, S.L., Plastino, A., 2008. Applications of the DM-GRASP heuristic: A survey. *International Transactions on Operational Research* 15, 387–416.
The authors survey opportunities on how to combine GRASP with data mining techniques.

Resende, M.G.C., Martí, R., Gallego, M., Duarte, A., 2008. GRASP and path relinking for the Max–Min diversity problem. *Computers and Operations Research.*
This paper proposes a GRASP with path relinking heuristic for finding approximate solutions to the Max-Min diversity problem. The heuristic hybridizes GRASP and path relinking, including GRASP with evolutionary path relinking variant. Empirical results show that the proposed hybrid implementations compare favorably to previous metaheuristics, such as tabu search and simulated annealing. To appear.

Nascimento, M.C.V., Resende, M.G.C., Toledo, F.M.B., 2008. GRASP with path-relinking for the multi-plant capacitated lot sizing problem. *European Journal of Operational Research.*
This paper describes a GRASP with path-relinking for the independent multi-plant, multi-period, and multi-item capacitated lot sizing problem where transfers between the plants are allowed. To appear.

5. Parallel GRASP

GRASP can be easily implemented in parallel by dividing iterations among several processors. Other parallelization schemes have also been used to implement parallel GRASPs. The following papers exemplify parallel GRASP.

Feo, T.A., Resende, M.G.C., Smith, S.H., 1994. A greedy randomized adaptive search procedure for maximum independent set. *Operations Research* 42, 860–878.
A GRASP for approximately solving the maximum independent set problem is described. The proposed heuristic can be easily implemented in parallel by decomposing the problem into smaller subproblems, each defined by conditioning on vertices being in the solution. An implementation of this algorithm was tested on a MIMD computer with up to eight processors. Average linear speedup is observed.

Pardalos, P.M., Pitsoulis, L., Mavridou, T., Resende, M.G.C., 1995. Parallel search for combinatorial optimization: Genetic algorithms, simulated annealing and GRASP. In Ferreira, A., Rolim, J. (eds) *Parallel Algorithms for Irregularly Structured Problems, Proceedings of the Second International Workshop – Irregular'95*, Vol. 980 of *Lecture Notes in Computer Science.* Springer-Verlag, Berlin, pp. 317–331.

This paper summarizes some parallel search techniques for approximating the global optimal solution of combinatorial optimization problems. For large-scale problems, one of the main limitations of heuristic search is its computational complexity. Therefore, efficient parallel implementation of search algorithms can significantly increase the size of the problems that can be solved.

Pardalos, P.M., Pitsoulis, L.S., Resende, M.G.C., 1995. A parallel GRASP implementation for the quadratic assignment problem. In Ferreira, A., Rolim, J. (eds) *Parallel Algorithms for Irregularly Structured Problems – Irregular'94*. Kluwer Academic Publishers, Dordrecht, MA, pp. 115–130.
Efficient parallel techniques for large-scale sparse quadratic assignment problems are discussed. The paper provides a detailed description of a parallel implementation on an MIMD computer of the sequential GRASP proposed by Li et al. (1994) for solving the QAP. The GRASP iterations are distributed among the processors. Each processor is given its own input data and random number sequence and the processors are run independently. A shared global variable stores the value of the incumbent solution.

Pardalos, P.M., Pitsoulis, L.S., Resende, M.G.C., 1996. A parallel GRASP for MAX-SAT problems. *Lecture Notes in Computer Science* 1184, 575–585.
A parallel GRASP for weighted maximum satisfiability (MAX-SAT) problem is proposed. The GRASP is based on the serial GRASP presented by Resende et al. (1997). The parallel implementation distributes the GRASP iterations among several processors operating in parallel, avoiding that two processors have as input the same random number generator seed. The best solution found among all processors is identified and used as solution of the problem.

Alvim, A.C.F., 1998. Parallelization strategies for the metaheuristic GRASP. Master's thesis, Department of Computer Science, Catholic University of Rio de Janeiro, Rio de Janeiro, RJ 22453-900 Brazil.
Two parallelization strategies for GRASP are discussed and compared: parallelization by distributing GRASP iterations and parallelization by varying the GRASP random parameter α. Both strategies are adapted to several parallel computation models, such as MPI (Message Passing Interface) and PVM (Parallel Virtual Machine). In Portuguese.

Alvim, A.C.F., Ribeiro, C.C., 1998. Load balancing in the parallelization of the metaheuristic GRASP. In *Tenth Brazilian Symposium of Computer Architecture*. Brazilian Computer Society, pp. 279–282.
Two parallelization strategies for GRASP are compared. The difference between the two strategies concerns the way in which data is partitioned: pre-scheduled (static load balancing) or self-scheduled (dynamic load balancing). The strategies have been tested considering an application to optimal traffic assignment in TDMA satellite system. Best results have been obtained by using the self-scheduling strategy. In Portuguese.

Martins, S.L., Ribeiro, C.C., Souza, M.C., 1998. A parallel GRASP for the Steiner problem in graphs. In Ferreira, A., Rolim, J. (eds) *Proceedings of IRREGULAR'98 – 5th International*

Symposium on Solving Irregularly Structured Problems in Parallel, Vol. 1457 of *Lecture Notes in Computer Science*. Springer-Verlag, Berlin, pp. 285–297.
A parallelization of a sequential GRASP for the Steiner minimal tree problem is proposed. The procedure implemented is one of the procedures described in Martins et al. (1999). The parallelization is accomplished by distributing the GRASP iterations among the processors on a demand-driven basis.

Murphey, R.A., Pardalos, P.M., Pitsoulis, L.S., 1998. A parallel GRASP for the data association multidimensional assignment problem. In Pardalos, P.M. (ed.) *Parallel Processing of Discrete Problems*, Vol. 106 of *The IMA Volumes in Mathematics and Its Applications*. Springer-Verlag, Berlin, pp. 159–180.
A parallel GRASP for finding good solutions for the data association problem is described.

Pardalos, P.M., Resende, M.G.C., Rappe, J., 1998. An exact parallel algorithm for the maximum clique problem. In De Leone, R. et al. (eds) *High Performance Algorithms and Software in Nonlinear Optimization*. Kluwer Academic Publishers, Dordrecht, MA, pp. 279–300.
A parallelized version of the exact algorithm of Carraghan and Pardalos (1990) for the unweighted maximum clique problem is described. A variant of the GRASP for the maximum independent set problem of Feo et al. (1994) is used for computing feasible solutions.

Rivera, L.I.D., 1998. Evaluation of parallel implementations of heuristics for the course scheduling problem. Master's thesis, Instituto Tecnologico y de Estudios Superiores de Monterrey, Monterrey, Mexico.
This thesis presents several parallel implementations of heuristics for the course scheduling problem. One of the heuristics is a GRASP. In Spanish.

Martins, S.L., 1999. Parallelization strategies for metaheuristics in distributed memory environments. Ph.D. thesis, Department of Computer Sciences, Catholic University of Rio de Janeiro, Rio de Janeiro, Brazil.
This thesis considers parallelization strategies for metaheuristics in distributed memory environments. GRASPs for the Steiner tree problem in graphs are described and implemented in parallel. In Portuguese.

Martins, S.L., Resende, M.G.C., Ribeiro, C.C., Pardalos, P.M., 2000. A parallel GRASP for the Steiner tree problem in graphs using a hybrid local search strategy. *Journal of Global Optimization* 17, 267–283.
A parallel GRASP for the Steiner problem in graphs is described.

Aiex, R.M., Resende, M.G.C., Ribeiro, C.C., 2002. Probability distribution of solution time in GRASP: An experimental investigation. *Journal of Heuristics* 8, 343–373.
The authors study the probability distributions of solution time to a solution with a given target solution value in five GRASPs that have appeared in the literature and for which source code is available. The distributions are estimated by running 12,000 independent runs of the heuristic. Standard methodology for graphical analysis is used to compare the empirical and theoretical distributions and estimate the parameters of the distributions. They

conclude that the solution time to a sub-optimal target value fits a two-parameter exponential distribution. Hence, it is possible to approximately achieve linear speed-up by implementing GRASP in parallel.

Duni Ekisoglu, S., Pardalos, P.M., Resende, M.G.C., 2002. Parallel metaheuristics for combinatorial optimization. In Corrêa, R. et al. (ed.) *Models for Parallel and Distributed Computation – Theory, Algorithmic Techniques and Applications*. Kluwer Academic Publishers, Dordrecht, MA, pp. 179–206.
Parallel metaheuristics for approximating hard combinatorial optimization problems are reviewed. Recent developments of parallel implementation of genetic algorithms, simulated annealing, tabu search, variable neighborhood search, and GRASP are discussed.

Ribeiro, C.C., Rosseti, I., 2002. A parallel GRASP heuristic for the 2-path network design problem. In *Proceedings of Euro-Par 2002*, Vol. 2400 of *Lecture Notes in Computer Science*. Springer, Berlin, pp. 922–926.
The authors propose a parallel cooperative strategy for GRASP with path relinking for the two-path network design problem. At each construction phase iteration a pair of nodes is selected at random and a shortest path connecting them is computed. Neighbors of a solution S are obtained by replacing in S any of its two-paths by another two-path between the same origin-destination pair.

Aiex, R.M., 2002. An experimental investigation of the probability distribution of solution time in GRASP heuristics and its application to the analysis of parallel implementations. Ph.D. thesis, Computer Science Department, Catholic University of Rio de Janeiro.
This thesis establishes that the running time of GRASP fits a shifted exponential distribution. Approximate linear speedup can therefore be achieved in parallel implementations of GRASP. The thesis also observes that GRASP with path-relinking has a run time distribution that approximately fits a shifted exponential. Parallel GRASP with path-relinking implementations for three-index assignment and job shop scheduling are described and tested. In Portuguese.

Aiex, R.M., Binato, S., Resende, M.G.C., 2003. Parallel GRASP with path-relinking for job shop scheduling. *Parallel Computing* 29, 393–430.
This paper describes a parallel GRASP with path relinking. Independent and cooperative parallelization strategies are described and implemented. Two greedy functions are defined. One of them takes into account makespan resulting from the inclusion of a candidate operation to the already-scheduled operations, while the second one (used in conjunction with the makespan) favors operations from jobs having long remaining processing times. The RCL is built by applying a min-max α-percentage rule. The authors employ the two-exchange local search.

Aiex, R.M., Resende, M.G.C., 2005. Parallel strategies for GRASP with path-relinking. In Ibaraki, T., Nonobe, K., Yagiura, M. (eds) *Metaheuristics: Progress as Real Problem Solvers*. Springer, Berlin.
The authors analyze two parallel strategies for GRASP with path-relinking and propose a criterion to predict parallel speedup based on experiments with a sequential implementation of the

algorithm. Independent and cooperative parallel strategies are described and implemented for the three-index assignment problem and the job-shop scheduling problem.

Ribeiro, C.C., Rosseti, I., 2007. Efficient parallel cooperative implementations of GRASP heuristics. *Parallel Computing* 33, 21–35.

This paper proposes and analyzes parallel cooperative strategies for GRASP with path-relinking for the two-path network design problem. At each construction iteration, a pair of nodes is selected at random and a shortest path connecting them is computed. Neighbors of a solution *S* are obtained by replacing in *S* any of its two-paths by another two-path between the same origin-destination pair.

Aiex, R., Resende, M.G.C., Pardalos, P.M., Toraldo, G., 2005. GRASP with path relinking for three-index assignment. *INFORMS Journal on Computing* 170, 2, 224–247.

This paper describes variants of GRASP with path relinking for the three-index assignment problem (AP3). Computational results show clearly that this GRASP for AP3 benefits from path relinking and that the variants considered in this paper compare well with previously proposed heuristics for this problem. The authors also show that the random variable *time to target solution* for all proposed GRASP with path relinking variants, fits a two-parameter exponential distribution. To illustrate the consequence of this, one of the variants of GRASP with path relinking is shown to benefit from parallelization.

6. Source code

Several papers describe computer source code implementing GRASP. The following papers all fall into this category.

Resende, M.G.C., Pardalos, P.M., Li, Y., 1996 Algorithm 754: Fortran subroutines for approximate solution of dense quadratic assignment problems using GRASP. *ACM Transactions on Mathematical Software* 22, 104–118.

This paper describes a set of ANSI standard Fortran 77 subroutines to find approximate solutions to dense quadratic assignment problems having at least one symmetric flow or distance matrix. It is an optimized implementation of the algorithm described in Li et al. (1994).

Pardalos, P.M., Pitsoulis, L.S., Resende, M.G.C., 1997. Algorithm 769: Fortran subroutines for approximate solution of sparse quadratic assignment problems using GRASP. *ACM Transactions on Mathematical Software* 23, 196–208.

A version of the GRASP for the quadratic assignment problem of Li et al. (1994), tailored for sparse instances is proposed. A set of ANSI standard Fortran 77 subroutines is described.

Resende, M.G.C., Feo, T.A., Smith, S.H., 1998. Algorithm 787: Fortran subroutines for approximate solution of maximum independent set problems using GRASP. *ACM Transactions on Mathematical Software* 24, 386–394.

This article describes a set of ANSI standard Fortran 77 subroutines to find an approximate solution of a maximum independent set problem. The GRASP used to produce the solutions is described in Feo et al. (1994).

Ribeiro, C.C., Resende, M.G.C., 1999. Algorithm 797: Fortran subroutines for approximate solution of graph planarization problems using GRASP. *ACM Transactions on Mathematical Software* 25, 341–352.
This paper describes a set of Fortran subroutines that implements the GRASP for graph planarization of Resende and Ribeiro (1997).

Resende, M.G.C., Pitsoulis, L.S., Pardalos, P.M., 2000. Fortran subroutines for computing approximate solutions of MAX-SAT problems using GRASP. *Discrete Applied Mathematics* 100, 95–113.
A set of Fortran subroutines for computing approximate solutions of MAX-SAT problems is described.

Festa, P., Pardalos, P.M., Resende, M.G.C., 2001. Algorithm 815: FORTRAN subroutines for computing approximate solution to feedback set problems using GRASP. *ACM Transactions on Mathematical Software* 27, 456–464.
A set of ANSI standard Fortran 77 subroutines for approximately solving the feedback vertex and arc set problems is described.

Aiex, R.M., Resende, M.G.C., Ribeiro, C.C., 2007. TTTPLOTS: A perl program to create time-to-target plots. *Optimization Letters* 1, 355–366.
This paper describes a Perl language program to create time-to-target solution value plots for measured CPU times that are assumed to fit a shifted exponential distribution, as in the case of randomized local search based heuristics for combinatorial optimization. The authors show how to use such plots in the comparison of different algorithms or strategies for solving a given problem. A detailed description of the Perl program tttplots.pl is also provided.