An Agent Simulation Model for the Québec Forest Supply Chain

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Abstract. A supply chain is a network of companies producing and distributing products to end-consumers. The Québec Wood Supply Game (QWSG) is a board game designed to teach supply chain dynamics. The QWSG provides the agent model for every company in our simulation. The goal of this paper is to introduce this simulation model. For this purpose, we first outline the QWSG, and then describe with mathematical equations each company in our simulation. Finally, three examples illustrate the use of our simulation to study collaboration in supply chains. More precisely, we study incentives for collaboration at both the supply chain and company level.

1 Introduction

Sterman (1989)'s Beer Game is a board-game designed to teach supply chain dynamics, and in particular, a problem called the bullwhip effect which is the amplification of order variability in supply chains (a supply chain is the network of companies producing and distributing products to end-customers). The Québec Wood Supply Game (QWSG) is an adaptation of the Beer Game to the Québec forest industry. Precisely, the QWSG has a diverging material flow structure, while the Beer Game has a straight one. This diverging flow begins at the output of the Sawmill, because this company has two clients, while all other companies only have one client, as illustrated by Figure 1. This diverging flow forces us to adapt the standard company model for the Sawmill. In fact, all companies have the same model as in the Beer Game, except the Sawmill which is modelled as two subcompanies sharing some parts, e.g., incoming transport. The QWSG is introduced in Section 2.

Kimbrough *et al.* (2002) replace human players by intelligent agents in the Beer Game. This modelization approach is interesting, because intelligent agents

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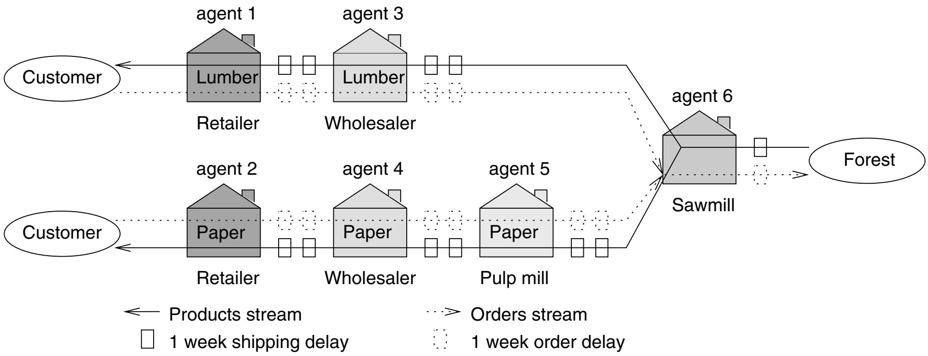

Fig. 1. Model of forest supply chain in the Québec Wood Supply Game

and human players/real companies share several properties, such as their autonomy, their reactive and proactive behaviours and their social ability (based on Wooldridge (2001)'s definition of an agent). We have a similar approach, except that we use the supply chain structure of the QWSG. We use the QWSG in order to replace human players by intelligent agents. Our goal is to study how local decisions taken by companies, impact global supply chain behaviour. Precisely, we give each company-agent a specific behaviour represented by its ordering scheme, and the simulation shows how this behaviour impacts the supply chain. In particular, companies can use ordering schemes which support information sharing, i.e., companies can collaborate in their ordering process. The notations, the model used for each company in the QWSG, and how to adapt this model to the Sawmill are presented in Section 3.

The Beer Game and the QWSG share a common method for cost evaluation. This is based on inventory holding and on backorder costs (backorders are incurred by stockouts, i.e., negative inventoy levels, and represent products that have to be shipped as soon as possible). We have adapted these costs to represent costs of a real Québec supply chain. Each company's method of cost calculation is presented in Section 4.

Finally, three research questions related to collaboration in supply chains illustrate this paper. The first question deals with the validation of a coordination mechanism improving the overall efficiency of the supply chain. The second question focusses on the Sawmill's incentive to behave according to either its own welfare, or overall supply chain welfare. Finally, in the third question, the use of Game Theory allows to extend the second question to every company. These three illustrations are presented in Section 5.

2 The Québec Wood Supply Game (QWSG)

Two board-games, called "Wood Supply Games", based on the structure and dynamics of Sterman (1989)'s Beer Game, were developped by Fjeld (2001) and Haartveit and Fjeld (2002). These games are an exercise that simulates the material and information flows in a production-distribution system, and were designed to make human players aware of the bullwhip effect. They illustrate the complex dynamics at the level of the supply chain, while each company has a simple model. Compared to the classic Beer Game that has been used to study supply chain dynamics, the Wood Supply Games introduce divergent product flows to increase its relevance to the North European forest sector. According to Fjeld (2001), such bifurcation represents the broad variety of products (paper, books, paperboard boxes, furniture, buildings...) manufactured from a few types of raw materials (wood). Our team, FOR@C (2003), has adapted this game for the Québec forest sector; we use this version, which is displayed in Figure 1. The main difference between the original and our Québec Wood Supply Game is in the length of the lumber and paper chain which is either the same (Fjeld's game) or different (our game), which corresponds to differences between Québec and Northern European wood industries.

Figure 1 shows how six players (human or software agents) play the QWSG. The game is played by turns: each turn represents a week in reality and is played in five steps; these five steps are played in parallel by each player. In the first step, players receive their inventory (these products were sent two weeks earlier by their supplier, because there is a two-week shipping delay) and advance shipping delays between suppliers and their customers. Then in the second step, players look at their incoming orders and try to fill them. If they have backorders, they try to fill those as well. If they do not have enough inventory, they ship as much as they can, and add the rest to their backorders. In the third step, players record their inventory or backorders. In the fourth step, players advance the order slips. In the last step, players place an order to their supplier(s), and record this order. To decide the amount to order, players compare their incoming orders with their inventory/backorder level (in our simulation, company-agent only evaluate an order scheme in a reactive way). The *correct decision* that would reduce the bullwhip effect has to be taken here. Finally, a new week begins with a new step 1, and so on.

Each position is played in the same way, except the Sawmill: this position receives two orders (one from the Lumber Wholesaler, another from the PulpMill) that have to be aggregated when placing an order to the Forest. The Sawmill can evaluate its order by basing it on the lumber demand or on the paper demand. In other words, the bifurcation in the supply chain makes the decision making harder by the PulpMill. Moreover, the Sawmill receives one type of product and each unit of this product generates two units: a lumber and a paper unit. That is, each incoming unit is split in two: one piece goes to the LumberSawmill's inventory, the other goes to the PaperSawmill's.

In our simulation, we add information sharing to the QWSG, as suggested by (Lee *et al.*, 1997; Moyaux *et al.*, 2003). Precisely, orders are explicit vectors (O, Θ), instead of a unique number $X = O + \Theta$, where O and Θ are hidden. In particular, O may be used to transmit the market consumption and Θ other company requirements (in our case, the fluctuation of inventory levels). Therefore, (O, Θ) orders are an information-based coordination mechanism (Boutilier, 1996; Wooldridge, 2001). We now present our simulation model.

3 Simulation Model Based on the QWSG

3.1 Notations

In order to share market demand, orders may have up to two dimensions, called O and Θ, as proposed in (Moyaux *et al.*, 2003). O placed in week w by company i is noted Op^i_w, and the corresponding Θ is noted Θp^i_w. The way to calculate these two quantities depends on the company-agent's behaviour, that is, on its ordering scheme. The other variables used in our simulation are:

To^i_w = company i's *o*utgoing *T*ransport in week w.
Too^i_w = company i's *o*utgoing *T*ransport in week w corresp. to current O.
Tob^i_w = company i's *o*utgoing *T*ransport in week w corresp. to current and backordered O.
$To\Theta^i_w$ = company i's *o*utgoing *T*ransport in w corresp. to backordered Θ.
Ti^i_w = company i's *i*ncoming *T*ransport in week w.
I^i_w = company i's *I*nventory in week w.
Op^i_w = company i's *P*laced *O*rders X in week w.
Oo^i_w = company i's *o*utgoing *O*rders O in week w.
Oi^i_w = company i's *i*ncoming *O*rders O in week w.
Ob^i_w = company i's *b*ackordered O in week w.
Θp^i_w = company i's sent Θ in week w.
Θo^i_w = company i's *o*utgoing Θ in week w.
Θi^i_w = company i's *i*ncoming Θ in week w.
Θb^i_w = company i's *b*ackordered Θ in week w.
$D^{\text{lumber}}_w = Oi^1_w$ = lumber market consumption in Week w.
$D^{\text{paper}}_w = Oi^2_w$ = paper market consumption in Week w.

Figure 2 presents some of (but not all) the relations between variables. We now further present these relations.

3.2 Company Model

The equations presented in this subsection hold for each company (except Sawmill that is described at the end of this subsection) using any ordering scheme. In fact,

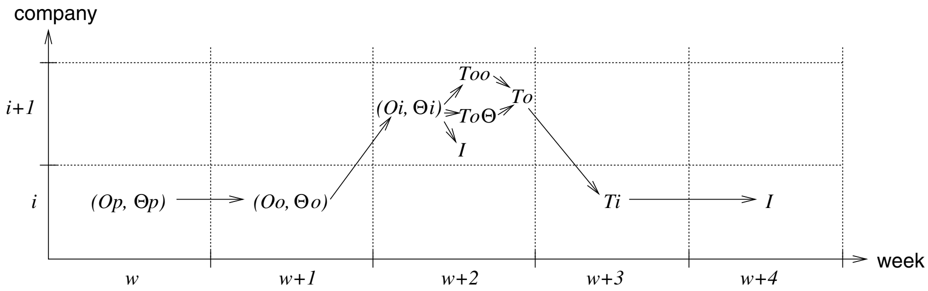

Fig. 2. Some relations between variables

here, we present the "mechanical" part of the QWSG, but the decision process will be presented in Section 5. As illustrated in Figure 2, an order $(Op, \Theta p)$ is placed in Week w by a client i. This order is put in Week $w+1$ in $(Oo, \Theta o)$ to represent a first week of ordering delay. To represent the second week of delay, this order is received by supplier $i+1$ in Week $w+2$ in $(Oi, \Theta i)$. This order reception decreases supplier's inventory I in the same week, because products are shipped in Too and $To\Theta$, and thus in To. In the next week, these shipped products are put in Week $w+3$ in client's Ti to represent a first week of shipping delay. To model the second week of shipping delay, these products are put in client's inventory in Week $w+4$. Therefore, to implement a two-week delay in information transmission and in transportation, order and transport variables are doubled, which explains the existence of pairs To_w^i/Ti_w^i and $(Oo_w^i, \Theta o_w^i)/(Oi_w^i, \Theta i_w^i)$. The simulation begins in Week $w=1$ and ends in $w=50$. In this section, for simplifying equations, $i=1$ represents a retailer (any of both retailers) and $i+1$ is i's supplier, such as $i=2$ is a wholesaler, even if this is not compatible with Figure 1. Therefore, Oi_w^1 denotes retailer's demand, that is, the market consumption (D_w^{lumber} or D_w^{paper}), but $Oo_w^1 = D_w^{\text{lumber}}$ and $Oo_w^2 = D_w^{\text{paper}}$ in the rest of this paper. Relations between variables that do not depend on the used ordering scheme are the same as in (Moyaux *et al.*, 2003, 2004; Moyaux, 2004).

To_w^n represents products sent to its client by company i in week w. To make the calculation of this quantity easy, it is divided into three parts such as $To_w^i = Too_w^i + Tob_w^i + To\Theta_w^i$. Too_w^i represents products that are first sent to fulfill the current order (or the quantity of products the company is able to ship when inventory and incoming transports are not enough), as reflected by Equation 1. Then, company i ships the quantity Tob_w^i of products to reduce its backorders (Equation 2). Finally, when orders are fulfilled and there is no backorder left, $To\Theta_w^i$ products are sent to reduce backordered Θ, called Θb_w^i (Equation 3).

$$Too_w^i = \begin{cases} Oi_w^i & \text{if } I_{w-1}^i \geq 0 \text{ and } I_{w-1}^i + Ti_w^i \geq Oi_w^i \\ I_{w-1}^i + Ti_w^i & \text{if } I_{w-1}^i \geq 0 \text{ and } I_{w-1}^i + Ti_w^i < Oi_w^i \\ Oi_w^i & \text{if } I_{w-1}^i < 0 \text{ and } Ti_w^i \geq Oi_w^i \\ Ti_w^i & \text{if } I_{w-1}^i < 0 \text{ and } Ti_w^i < Oi_w^i \end{cases} \tag{1}$$

$$Tob_w^i = \begin{cases} Ob_{w-1}^i & \text{if } I_{w-1}^i \geq 0 \text{ and } I_{w-1}^i \\ & +Ti_w^i - Too_w^i \geq Ob_{w-1}^i \\ I_{w-1}^i + Ti_w^i - Too_w^i & \text{if } I_{w-1}^i \geq 0 \text{ and } I_{w-1}^i \\ & +Ti_w^i - Too_w^i < Ob_{w-1}^i \\ -I_{w-1}^i & \text{if } I_{w-1}^i < 0 \text{ and } Ti_w^i - Too_w^i \geq -I_{w-1}^i \\ Ti_w^i - Too_w^i & \text{if } I_{w-1}^i < 0 \text{ and } Ti_w^i - Too_w^i < -I_{w-1}^i \end{cases} \tag{2}$$

$$ To\Theta_w^i = \begin{cases} -Too_w^i - Tob_w^i & \text{if } I_{w-1}^i \geq 0 \text{ and } I_{w-1}^i + Ti_w^i - Too_w^i \\ & -Tob_w^i \geq \Theta b_{w-1}^i + \Theta i_w^i \text{ and } Too_w^i \\ & +Tob_w^i + \Theta b_{w-1}^i + \Theta i_w^i < 0 \\ \Theta b_{w-1}^i + \Theta i_w^i & \text{if } I_{w-1}^i \geq 0 \text{ and } I_{w-1}^i + Ti_w^i - Too_w^i \\ & -Tob_w^i \geq \Theta b_{w-1}^i + \Theta i_w^i \text{ and } Too_w^i \\ & +Tob_w^i + \Theta b_{w-1}^i + \Theta i_w^i \geq 0 \\ I_{w-1}^i + Ti_w^i - Too_w^i - Tob_w^i & \text{if } I_{w-1}^i \geq 0 \text{ and } I_{w-1}^i + Ti_w^i - Too_w^i \\ & -Tob_w^i < \Theta b_{w-1}^i + \Theta i_w^i \\ -Too_w^i - Tob_w^i & \text{if } I_{w-1}^i < 0 \text{ and } Ti_w^i - Too_w^i - Tob_w^i \\ & \geq \Theta b_{w-1}^i + \Theta i_w^i \text{ and } Too_w^i + Tob_w^i \\ & +\Theta b_{w-1}^i + \Theta i_w^i < 0 \\ \Theta b_{w-1}^i + \Theta i_w^i & \text{if } I_{w-1}^i < 0 \text{ and } Ti_w^i - Too_w^i - Tob_w^i \\ & \geq \Theta b_{w-1}^i + \Theta i_w^i \text{ and } Too_w^i + Tob_w^i \\ & +\Theta b_{w-1}^i + \Theta i_w^i \geq 0 \\ Ti_w^i - Too_w^i - Tob_w^i & \text{if } I_{w-1}^i < 0 \text{ and } Ti_w^i - Too_w^i - Tob_w^i \\ & < \Theta b_{w-1}^i + \Theta i_w^i \end{cases} \tag{3} $$

Backorders correspond to products needed by clients, but that cannot currently be shipped. Their role is to represent products that should have been shipped in the past or in the current week, and that have to be shipped as soon as possible. Like orders, backorders are represented by two variables: Ob_w^i in Equation 4 represents backorders created by unfulfilled O, and Θb_w^i in Equation 5 backorders created by unfulfilled Θ.

$$ Ob_w^i = Ob_{w-1}^i + (Oi_w^i - Too_w^i) - Tob_w^i \tag{4} $$

$$ \Theta b_w^i = \begin{cases} \Theta b_{w-1}^i + \Theta i_w^i - To\Theta_w^i + Too_w^i + Tob_w^i + To\Theta_w^i & \text{if } Too_w^i + Tob_w^i \\ & +To\Theta_w^i < 0 \\ \Theta b_{w-1}^i + \Theta i_w^i - To\Theta_w^i & \text{if } Too_w^i + Tob_w^i \\ & +To\Theta_w^i \geq 0 \end{cases} \tag{5} $$

Incoming transport is supplier $i+1$'s last week outgoing transport:

$$ Ti_w^i = To_{w-1}^{i+1} \tag{6} $$

Inventory level is previous inventory level plus inputs minus outputs:

$$ I_w^i = I_{w-1}^i + Ti_w^i - To_w^i \tag{7} $$

Figure 2 shows how orders are delayed between a client i and its supplier $i+1$. Each order is placed in $(Op_w^i, \Theta p_w^i)$, goes next in $(Oo_{w+1}^i, \Theta o_{w+1}^i)$ to simulate the first week of delay, and is finally put in supplier's $(Oi_{w+2}^{i+1}, \Theta i_{w+2}^{i+1})$ to simulate the second week of delay. This explains why incoming order (O, Θ) is the last week client $i-1$'s outgoing transport (Equations 8 and 9).

$$ Oi_w^i = Oo_{w-1}^{i-1} \tag{8} $$

$$\Theta i_w^i = \Theta o_{w-1}^{i-1} \tag{9}$$

Figure 2 also explains how Oo_w^i and Θo_w^i are setup in Equations 10 and 11.

$$Oo_w^i = Op_{w-1}^i \tag{10}$$

$$\Theta o_w^i = \Theta p_{w-1}^i \tag{11}$$

Every company is setup in the same way, except the SawMill which has to process two types of products: lumber and paper. As a consequence, some parts of this company are doubled to manage this particularity. Specifically, we use the pairs of variables $Oi_w^{6\text{-lumber}}$ / $Oi_w^{6\text{-paper}}$, $\Theta i_w^{6\text{-lumber}}$ / $\Theta i_w^{6\text{-paper}}$, $Op_w^{6\text{-lumber}}$ / $Op_w^{6\text{-paper}}$, $\Theta p_w^{6\text{-lumber}}$ / $\Theta p_w^{6\text{-paper}}$, $I_w^{6\text{-lumber}}$ / $I_w^{6\text{-paper}}$ and $To_w^{6\text{-lumber}}$ / $To_w^{6\text{-paper}}$. In addition, some additional variables Op_w^6 and Θp_w^6 handle the interface between these pairs of variables and the Sawmill's supplier. $Oo_w^6, \Theta o_w^6$ and Ti_w^6 are unique. In fact, we operate as if there were a LumberSawMill and a PaperSawMill having the same product input Ti_w^6 (each subcompany of the Sawmill receives what is in the incoming transport Ti_w^6 because one unit of wood coming from the Forest provides one unit of lumber and one unit of paper). Then, the rest of the LumberSawMill is distinct from the PaperSawMill. In particular, the LumberSawmill would like to place orders $(Op_w^{6\text{-lumber}}, \Theta p_w^{6\text{-lumber}})$, while the PaperSawmill prefers $(Op_w^{6\text{-paper}}, \Theta p_w^{6\text{-paper}})$. The design of the ordering schemes managing the variables $(Op_w^i, \Theta p_w^i)$ are outined in Section 5. We also compare in Subsection 5.2 four methods to aggregate $(Op_w^{6\text{-lumber}}, \Theta p_w^{6\text{-lumber}})$ and $(Op_w^{6\text{-paper}}, \Theta p_w^{6\text{-paper}})$ into $(Op_w^6, \Theta p_w^6)$. Before that, we present how costs in the QWSG are made realistic.

4 Cost Evaluation

Here, we adapt cost parameters from the Québec industry to the cost function in the QWSG. Precisely, the calculation of company i's cost C^i is adapted from the QWSG method, in which it is the sum of company i's inventory plus two times the sum of its backorders during the whole simulation: $C^i_{\text{board QWSG}} = \sum_{w=1}^{50} \{I_w^i + 2 * (Ob_w^i + \Theta b_w^i)\}$. This QWSG cost is the base that is next translated into real cost. First, one of two conversion factors is applied: (i) CF^{lumber} translates units in the QWSG into lumber quantity measured in Mpmp (i.e., one thousand pmp -in French "pied mesure planche"- where 1 pmp is a 1 inch × 1 foot × 1 foot piece of wood), and (ii) CF^{paper} translates simulated units into quantity of paper measured in tma (i.e., one anhydrous metric ton, which represents between 2.5 and 2.8 m^3 depending on the type of wood). Secondly, we consider a lumber market consumption of 70,000 Mpmp/year, and a paper market consumption of 63,000 tma/year, because the Sawmill has a transformation ratio of

0.9 tma / Mpmp[1]. Therefore, we use $CF_{\text{lumber}} = 70,000/\sum_{w=1}^{50} D_w^{\text{lumber}}$ and $CF_{\text{paper}} = 63,000/\sum_{w=1}^{50} D_w^{\text{paper}}$. For example, if $D_w^{\text{lumber}} = D_w^{\text{paper}} = 11$ for all Week w (we call "Step" this market consumption pattern), $\sum_{w=1}^{50} D_w^{\text{lumber}} = \sum_{w=1}^{50} D_w^{\text{paper}} = 11+11+\ldots+11 = 11*50 = 550$ units, and thus $CF^{\text{lumber}} = 70,000/550 = 127$ Mpmp/units and $CF^{\text{paper}} = 63,000/550 = 115$ tma/units, where units are the products simulated in the QWSG. Thirdly, we use sell prices to convert these quantities into Canadian dollars (CAD) by applying the prices $P^{\text{lumber}} = 430$ CAD / Mpmp, $P^{\text{6-paper}} = 125$ CAD / tma and $P^{\text{5-paper}} = 690$ CAD / tma (paper has two prices, because the PulpMill transforms it and thus increases its value). These parameters come from the Conseil de l'Industrie Forestière du Québec (2004)'s Pribec[2]. Finally, we apply a ratio of 37% representing logistic costs according to Nahmias (1997). This leads to costs in Equations 12, 13, 14, 15, 16 and 17.

$$C^1_{\text{realistic}} = CF_{\text{lumber}} * P^{\text{lumber}} * 0.37 * 1.37^2 * \sum_{w=1}^{50} \{I_w^1 + 2 * (Ob_w^1 + \Theta b_w^1)\} \quad (12)$$

$$C^2_{\text{realistic}} = CF_{\text{paper}} * P^{\text{5-paper}} * 0.37 * 1.37^2 * \sum_{w=1}^{50} \{I_w^2 + 2 * (Ob_w^2 + \Theta b_w^2)\} \quad (13)$$

$$C^3_{\text{realistic}} = CF_{\text{lumber}} * P^{\text{lumber}} * 0.37 * 1.37 * \sum_{w=1}^{50} \{I_w^3 + 2 * (Ob_w^3 + \Theta b_w^3)\} \quad (14)$$

$$C^4_{\text{realistic}} = CF_{\text{paper}} * P^{\text{5-paper}} * 0.37 * 1.37 * \sum_{w=1}^{50} \{I_w^4 + 2 * (Ob_w^4 + \Theta b_w^4)\} \quad (15)$$

$$C^5_{\text{realistic}} = CF_{\text{paper}} * P^{\text{5-paper}} * 0.37 * \sum_{w=1}^{50} \{I_w^5 + 2 * (Ob_w^5 + \Theta b_w^5)\} \quad (16)$$

$$\begin{aligned} C^6_{\text{realistic}} &= CF_{\text{lumber}} * P^{\text{lumber}} * 0.37 * \textstyle\sum_{w=1}^{50} \{I_w^{\text{6-lumber}} \\ &+2 * (Ob_w^{\text{6-lumber}} + \Theta b_w^{\text{6-lumber}})\} + CF_{\text{paper}} * P^{\text{6-paper}} \\ &*0.37 * \textstyle\sum_{w=1}^{50} \{I_w^{\text{6-paper}} + 2 * (Ob_w^{\text{6-paper}} + \Theta b_w^{\text{6-paper}})\} \end{aligned} \quad (17)$$

[1] This also represents a input of 315,000 m^3/year of wood coming from the Forest, because there is a transformation ratio of 4.5 m^3/Mpmp, but we do not use this information in our simulation.

[2] I thank Martin Cloutier (CRIQ -Québec Industrial Research Center- and Master's student in FOR@C (2003)) who kindly gave to me these parameters.

5 Study of Collaboration

The three following subsections illustrate how this simulation model and the previous method of cost evaluation can be used to study both the efficiency increase of the overall supply chain which is due to collaboration, and company's individual incentive for collaboration. The first subsection deals with the choice of a common ordering scheme for all companies, in which collaboration with information sharing should improve the supply chain efficiency. We call this type of supply chain "homogeneous", in which every company applies the same ordering scheme.

The second subsection focusses on the case of the Sawmill, which is the only company having two clients to satisfy. Here, the question is if the Sawmill should act for itself, or for the whole supply chain efficiency. Conversely to the two first subsections, the third subsection no longer assumes that the supply chain is homogeneous. Precisely, companies are allowed to use different ordering schemes, and concepts from Game Theory are adapted to our simulation model to find the ordering scheme each company prefers to use. Thus, we address a similar question than in the second subsection, but we extend it to the whole supply chain, instead of the single Sawmill. The question in Subsection 5.3 can also be stated as: "do companies prefer collaboration or selfish behaviour?"

5.1 Study of Decentralized Coordination Mechanisms

Our simulation first allows us to study the dynamics in a whole supply chain when this supply chain is homogeneous, that is, where all companies use the same ordering scheme. The goal here is to understand which ordering scheme produces an efficient supply chain. Precisely, we propose in (Moyaux *et al.*, 2003) two ordering schemes, called B and D, for reducing the bullwhip effect, i.e., for improving the overall supply chain efficiency, where orders are vectors (O, Θ), instead of a single number X. The use of this vector allows companies to share information in order to collaborate. In this case, collaboration aims at improving the supply chain efficiency by reducing the bullwhip effect. However, we do not insist in this paper how this reduction is obtained, Moyaux *et al.* (2003, 2004) detail this point. We only note that companies have to share information, i.e., they have to collaborate by agreeing to perform this sharing. Since our two Schemes B and D are compared with A and C, from which they are derived to show why and how to share demand information (Moyaux *et al.*, 2003), we now present these four schemes. Let us mention that B and D use (O, Θ) orders, and C and D use information centralization:

Scheme A does not use Θ (Equations 18).

$$\theta p_w^i = 0 \tag{18}$$

Orders are placed with A such as to keep a steady inventory, except that negative orders (i.e., order cancellations) are forbidden (Equation 19).

$$Op_w^i = \begin{cases} 0 & \text{if } Oi_w^i + (I_{w-1}^i - I_w^i) + (Ob_w^i - Ob_{w-1}^i) < 0 \\ Oi_w^i + (I_{w-1}^i - I_w^i) + (Ob_w^i - Ob_{w-1}^i) & \text{else} \end{cases} \tag{19}$$

Scheme B is the first ordering scheme proposed to reduce the bullwhip effect with (O, Θ) orders. Incoming O, which is the market consumption when all clients use Schemes B or D, is transmitted to the client (see Equation 20):

$$Op^i_w = Oi^i_w \tag{20}$$

Next, company requirements are added to incoming Θ and this sum is sent as Θ to the supplier (see Equation 21). $\lambda * (Oi^i_{w-1} - Oi^i_w)$ is an estimation of the inventory decrease caused by the variation of Oi, which is caused by the market consumption variation. λ is chosen such as the inventory eventually stabilizes on its initial level when market consumption is again steady after a variation. We take $\lambda = 4$ (see Moyaux (2004)'s thesis for details).

$$\theta p^i_w = \theta i^i_w - \lambda * (Oi^i_{w-1} - Oi^i_w) \tag{21}$$

The two next ordering schemes apply information centralization, in which retailers multicast the market consumption to the whole supply chain. Information sharing with information centralization is much quicker than information sharing with (O, Θ) orders, because market consumption transmitted in O is as slow as orders, while information centralization is assumed to be instantaneous and in real-time.

Scheme C does not use Θ, as reflected by Equation 22.

$$\Theta p^i_w = 0 \tag{22}$$

Next, C is similar to A, except that information centralization is used, like in D. Remember that $D^{\text{lumber}}_w = Oi^1_w$ and $D^{\text{paper}}_w = Oi^2_w$, depending on whether the considered company i works with lumber or paper. Therefore, Oi^i_w in Equation 19 is replaced by Oi^{retailer}_w when the retailer agrees to multi-cast market consumption, else, by $Oi^{\text{wholesaler}}$ when the retailer disagrees to multi-cast, but the wholesaler agrees, ... else, by Oi^i_w when no companies multicasts its incoming orders. We call k the first company agreeing to multi-cast its incoming orders in Equation 23:

$$Op^i_w = \begin{cases} 0 & \text{if } Oi^k_w + (I^i_{w-1} - I^i_w) + (Ob^i_w - Ob^i_{w-1}) < 0 \\ Oi^k_w + (I^i_{w-1} - I^i_w) + (Ob^i_w - Ob^i_{w-1}) & \text{else} \end{cases} \tag{23}$$

Scheme D is very similar to Scheme B. The difference is in the way to choose O and Θ: it is based on market consumption when information centralization is achieved by the retailer, else they are based on the wholesaler's incoming order when the retailer does not multi-cast his incoming order but the wholesaler does... else the company uses its own incoming order when none of its clients multi-cast its incoming orders. Like in Scheme C, k refers to the first company agreeing to multi-cast its incoming orders (Equation 24):

$$Op^i_w = Oi^k_w \tag{24}$$

In the same way, Oi^i_w in Equation 21 is replaced by Company k's incoming order, i.e., by the market consumption when the retailer agrees to multi-cast

Table 1. Standard-deviation of orders and Supply chain costs

	Scheme A	Scheme B	Scheme C	Scheme D
$\sigma_{Op^i_w+\Theta p^i_w}$	14.3; 13.3; 28.2; 26.5; 51; 42.6	3.8; 3.8; 7.2; 7.2; 10.6; 8.7	5.8; 5.4; 7.9 7.7; 7.1; 4.9	3.8; 3.8; 4.2 4.2; 4.5; 4.2
$C_{\text{realistic}}$	51,221+85,507 +123,521+161,963 +164,206+143,873 =730,291 k$	27,870+66,643 +18,120+46,093 +28,056+15,323 =202,105 k$	62,783+118,483 +49,996+113,199 +80,764+**34,776** =460,001 k$	17,666+52,004 +10,672+35,407 +21,886+10,538 =148,173 k$

it. We explain in Moyaux (2004)'s thesis that we have to take $\lambda = 2$ for every companies, except for retailers that have $\lambda = 4$ (Equation 25):

$$\theta p^i_w = \theta i^i_w - \lambda * (Oi^k_{w-1} - Oi^k_w) \tag{25}$$

The initial conditions are $I^i_0 = 0$ for every company i and for every ordering scheme. As previously stated, the demand Step indicates that market consumption in the QWSG is eleven products per week in the four first weeks, and then seventeen products until the end of the simulation ($D^{\text{lumber}}_w = D^{\text{paper}}_w = 11$ for $w \in \{1, 2, 3, 4\}$, and $D^{\text{lumber}}_w = D^{\text{paper}}_w = 17$ for $w = 5, 6, 7, \dots 50$). Since the bullwhip effect is the order variability, it is measured in Table 1 as the standard deviation of orders placed by companies. In this Table, the i^{th} number is the standard deviation of orders placed by Company i according to Figure 1, e.g., the PulpMill ($i = 5$) places orders with a standard deviation of 4.5 under Ordering Scheme D. Table 1 also presents individual costs, as introduced in Section 4, with the format $C^1_{\text{realistic}} + C^2_{\text{realistic}} + C^3_{\text{realistic}} + C^4_{\text{realistic}} + C^5_{\text{realistic}} + C^6_{\text{realistic}} = C_{\text{realistic}}$, e.g., the PaperRetailer $i = 2$ has a cost $C^2_{\text{realistic}} = 52,004$ k$ with D. The lowest values in Table 1 are underlined. For example, $C = 148,173$ k$ with Scheme D is underlined, because it is lower than $C = 730,291$ k$ incurred by A, and than $C = 202,107$ k$ incurred by B, and than $C = 460,001$ k$ incurred by C.

We can see that D has the best results for the whole supply chain, and also for every company. Let us recall that D is the highest level of collaboration, because it applies an improved form of collaboration called "information centralization". Our paper (Moyaux *et al.*, 2004) extends these results for eighteen other market consumption patterns and under three other ordering schemes. We only recall the results in Table 1 to illustrate the possible uses of our simulation to study collaboration in supply chains. On the contrary, the next subsection presents *unpublished* results. We now question whether the Sawmill could change the way it places orders, in order to reduce its individual cost $C^6_{\text{realistic}}$.

5.2 The Case of the Sawmill

We have seen that the Sawmill is modelled as two subcompanies called LumberSawmill and PaperSawmill. Each subscompany is similar to other companies

Table 2. New Table 1 when the Sawmill aggregates its lumber and paper requirements with a mean function

	Scheme A	Scheme B	Scheme C	Scheme D
$\sigma_{Op^i_w+\Theta p^i_w}$	11.4; 12.5; 23.5 25.6; 50; 74.2	3.8; 3.8; 7.2; 7.2; 10.6; 17.1	8.1; 8.7; 9.9; 11.8; 11.1; 13.5	3.8; 3.8; 4.2; 4.2; 4.5; 5.6
$C_{\text{realistic}}$	52,237+87,401 +74,331+145,828 127,573+341,167 =828,537 k$	23,758+47,634 +15,118+31,579 +16,997+65,490 =**200,576** k$	54,876+108,377 +38,036+95,216 +55,628+38,677 =**390,810** k$	17,057+38,019 +10,227+24,562 +13,504+18,892 =122,261 k$

Table 3. New Table 1 when the Sawmill only takes care of its lumber requirements

	Scheme A	Scheme B	Scheme C	Scheme D
$\sigma_{Op^i_w+\Theta p^i_w}$	13.6; 13.6; 27; 26.3; 47.9; 50.4	3.8; 3.8; 7.2; 7.2; 10.6; 10.6	6.7; 6.4; 9; 9; 7.5; 6.7	3.8; 3.8; 4.2; 4.2; 4.5; 4.5
$C_{\text{realistic}}$	61,983+114,641 +83,873+119,352 +117,271+**111,236** =**608,356** k$	23,758+73,854 +15,118+51,994 +32,830+**11,160** =208,714 k$	65,486+131,102 +53,136+129,826 +96,626+40,936 =517,112 k$	17,057+64,240 +10,227+44,976 +29,337+**8,726** =174,563 k$

in the QWSG, except that it shares some elements with the other subcompany. This particularity of the Sawmill is required by the diverging material flow simulated in the QWSG. In addition to the questions related to company behaviour that are outlined in the previous subsection, another issue appears here. The LumberSawmill would like to place orders $(Op_w^{6\text{-lumber}}, \Theta p_w^{6\text{-lumber}})$, while the PaperSawmill prefers $(Op_w^{6\text{-paper}}, \Theta p_w^{6\text{-paper}})$. These two different orders are chosen according to the same scheme A, B, C or D as the rest of the supply chain (homogeneous supply chain). The question is how to aggregate these two orders so as to place only one order $(Op_w^6, \Theta p_w^6)$. Four aggregation methods are studied:

1. $(Op_w^6, \Theta p_w^6) = ((Op_w^{6-lumber} + Op_w^{6-paper})/2, (\Theta p_w^{6-lumber} + \Theta p_w^{6-paper})/2)$ was used in Subsection 5.1. Its results are thus in Table 1.
2. $(Op_w^6, \Theta p_w^6) = (\max(Op_w^{6-lumber}; Op_w^{6-paper}), \max(\Theta p_w^{6-lumber}; \Theta p_w^{6-paper}))$ is applied in (Moyaux *et al.*, 2003, 2004) to avoid backorders by the Sawmill, because this company orders at least (and often more than) what it needs. The incurred results are presented in Table 2.
3. $(Op_w^6, \Theta p_w^6) = (Op_w^{6-lumber}, \Theta p_w^{6-lumber})$, that is, the Sawmill assumes lumber is far much important for itself than paper. The standard deviation of placed orders and the costs are presented in Table 3.
4. $(Op_w^6, \Theta p_w^6) = (Op_w^{6-paper}, \Theta p_w^{6-paper})$, that is, the Sawmill assumes paper is far much important for itself than lumber. The incurred results are presented in Table 4.

The lowest $C_{\text{realistic}}$ and $C^6_{\text{realistic}}$ between Tables 1, 2, 3 and 4 are written in bold. For example, $C^6_{\text{realistic}} = 34,776$ k$ in Table 1 with Scheme C is bold,

Table 4. New Table 1 when the Sawmill only takes care of its paper requirements

	Scheme A	Scheme B	Scheme C	Scheme D
$\sigma_{Op^i_w+\Theta p^i_w}$	9.3; 10.9; 20.8; 22; 43.7; 87.3	3.8; 3.8; 7.2; 7.2; 10.6; 14	7.5; 6.5; 9.5; 9.6; 9.7; 6.3	3.8; 3.8; 4.2; 4.2; 4.5; 4.8
$C_{\text{realistic}}$	61,400+113,294 +93,359+144,685 +127,826+205,967 =746,531 k$	31,982+59,433 +21,121+40,192 +23,283+19,486 =195,497 k$	66,095+106,484 +55,359+97,396 +68,298+40,928 =434,560 k$	18,275+39,767 +11,116+25,838 +14,436+12,349 =**121,781** k$

because it is lower than $C^6_{\text{realistic}} = 38,677$ k$ in Table 2, than $C^6_{\text{realistic}} = 40,936$ k$ in Table 3, and than $C^6_{\text{realistic}} = 40,928$ k$ in Table 4. We can note that the Sawmill sometimes prefers an aggregation method which is best for itself, but not for the whole supply chain, i.e., for a given ordering scheme, $C^6_{\text{realistic}}$ written bold is not in the same table than $C_{\text{realistic}}$ written bold. In particular, when every company uses Scheme D, the Sawmill chooses the third aggregation method in Table 3, because it has the lowest cost $C^6_{\text{realistic}} = 8,726$ k$. While the rest of the supply chain would rather that the Sawmill uses the fourth aggregation method in Table 4, because the supply chain incurs cost $C_{\text{realistic}} = 121,781$ k$ (which is lower than $C_{\text{realistic}} = 174,563$ k$ in Table 3). This opposition between individual cost and overall supply chain cost is extended to the whole supply chain in Subsection 5.3.

5.3 Analysis of Agents' Incentives for Collaboration

In the two previous subsections, the supply chain was homogeneous. We relax this assumption now by allowing each company to choose an ordering scheme among three. The first two schemes are B and D. The third one is an implementation of the (s, S) policy, which is classic in Inventory Management (Nahmias, 1997).:

Scheme A" is an (s, S) ordering policy that does not use (O, Θ) orders. Therefore, incoming Θ orders are fulfilled by shipping items to the client as actual orders, as is done with all ordering rules, but no Θ are placed (Equation 26).

$$\theta p^i_w = 0 \tag{26}$$

Each week, the company checks if inventory is lower than s. When this occurs, the company orders products to fill its inventory up to S (see Equation 27). We show in (Moyaux, 2004)'s thesis that taking $s = 0$ and $S = Oi^i_w$ is optimal under two assumptions that are not met in our simulations: company demand is steady, and the company's supplier is an infinite source of products that never incurs backorders. We discuss in Moyaux (2004)'thesis the reason why it is very hard to relax these two assumptions.

$$Op^i_w = \begin{cases} S - I^i_w & \text{if } I^i_w < s \\ 0 & \text{else} \end{cases} = \begin{cases} Oi^i_w - I^i_w & \text{if } I^i_w < 0 \\ 0 & \text{else} \end{cases} \tag{27}$$

The question studied here is to know if companies prefer either not collaborating by using A", or partially collaborating by using B, or fully collaborating with D. In other words, is a company sacrificing itself for the welfare of the rest of the supply chain? It if this situation occurs, companies preferring collaboration would have to give money to this company, in order to incitate it to collaborate. In this case, the money transfer would have to be quantified.

This kind of questions concerning agents' preference is adressed with our simulation by carrying out many simulations. Specifically, there are $3^6 = 729$ simulations because we simulate six companies having each three different available ordering schemes, as reflected by Algorithm 1 (we can note here that a combinatorial explosion arises when companies are added). Precisely, all companies use the first scheme in the first simulation, next all companies use the first scheme, except a company using the second scheme..., and finally, all companies use the third scheme in the 729^{th} simulation. After each simulation, we note the individual cost $C^i_{\text{realistic}}$ for each company. The methodology to carry out all these simulations is outlined in Algorith 1. We have also included an optimization of the parameters of the ordering schemes, but we do not present that for the sake of simplicity.

Algorithm 1 Methodology to simulate every configuration of the supply chain

```
simulateSupplyChain()
returns sets of simulated costs {C^i_realistic}_{i=1}^{i=6}
for each r^1 ∈ {A", B, D} do
  set LumberRetailer to ordering scheme r^1
  for each r^2 ∈ {A", B, D} do
    set PaperRetailer to ordering scheme r^2
    for each r^3 ∈ {A", B, D} do
      set LumberWholesaler to ordering scheme r^3
      for each r^4 ∈ {A", B, D} do
        set PaperWholesaler to ordering scheme r^4
        for each r^5 ∈ {A", B, D} do
          set PulpMill to ordering scheme r^5
          for each r^6 ∈ {A", B, D} do
            set Sawmill to ordering scheme r^6
            simulate the supply chain under Consumption Step
            save the simulated values of {C^i_realistic}_{i=1}^{i=6}
```

Each simulation produces a set of costs $\{C^i_{\text{realistic}}\}_{i=1}^{i=6}$. Therefore, each simulation produces a set $\{C^i_{\text{realistic}}\}_{i=1}^{i=6}$ to fill one of the $3^6 = 729$ entries of a game in the normal form, according to Game Theory. The game built with these $6 * 729$ outcomes is next analyzed by Gambit 0.97.05. Gambit is a free software from the Gambit Project (2003), and is licensed under the Free Software Foundation (2004)'s GNU General Public License for analyzing games according to Game Theory principles. The result of the analysis of this game are as follows:

– The *minimum of overall supply chain cost* C is incurred when every company uses D. This means that using D is the best solution for the overall supply chain, as stated in Subsection 5.1. Conversely, to Subsection 5.1, it could have ocurred, for example, that cost C is minimum when a part of the supply chain uses D, and the other part B. This is not the case: the homogeneous supply chain is the best solution here from a global point of view.
– There is only one *Nash equilibrium*, which ocurs when every company uses D. This means that no companies has an incentive to deviate unilaterally from D.

If we consider these two results together, we see that full collaboration of the whole supply chain is both the best choice for the overall supply chain, and for each company in this supply chain. In fact, if one company unilaterally stops using D, i.e., if it stops fully collaborating, it increases its costs $C^i_{\text{realistic}}$, because it leaves a Nash equilibrium.

6 Conclusion

This paper has presented the QWSG which is a board-game designed to teach supply chain dynamics. This game provides an agent modelisation of each company in a supply chain. We implemented this game and equations describing each company are presented in this paper. Next, we show how we adapt costs from the Québec forest industry to our simulation. Finally, three examples illustrate some possible uses of this simulation. The first example compares two collaboration-based ordering schemes with two other schemes. We verify here that collaboration is advantageous for the overall supply chain. Nevertheless, collaboration could still be disadvantageous for one company. We first establish that choices taken by a particular company in our simulation, the Sawmill, may be good for the whole supply chain, while it is bad for this company, or the contrary. Similarly, we extend this kind of question to every company. Here, we use Game Theory to check that the problem experienced by the Sawmill does not arise with every company. Precisely, companies can choose one of three levels of collaboration (no collaboration, partial collaboration and full collaboration). In this context, the only Nash equilibrium is when every company fully collaborates. In addition, this equilibrium also incurs the lowest overall supply chain cost. As a consequence, *companies should fully collaborate.*

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